Topological phenomena in quantum physics and applications

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1 General context

In recent years, there has been a growing interest in quantum systems exhibiting non-trivial topological properties, such as Chern classes of fiber bundles. One remarkable consequence is that emergent experimental properties are robust against perturbations. For instance, the integer quantum Hall effect involves the first Chern class of a fiber bundle over the Brillouin zone and manifests as an electrical conductivity quantized in integer multiples of e^2/h with extremely high precision. Moreover, topological qubits for quantum computing have attracted significant interest [3].

In some cases, the theoretical description is subtle and complex, involving the Atiyah-Singer index theorem, a major result of the 20th century that bridges spectral theory and algebraic topology via microlocal analysis [1]. This framework helps explain the presence of discrete states in spectral gaps and the spectral flow of a finite number of quantum states [4]. These quantum states sometimes appear as bound states or robust edge current states.

Current physical models are limited to symmetric cases and do not fully exploit the power of microlocal analysis. Establishing a generic model based on microlocal analysis and the index theorem remains a key challenge in the field.

2 Objectives of the thesis

This thesis will be conducted in mathematical physics **at the Institut Fourier** in collaboration with leading physicists in the field : Hong Qin (princeton PPL), Jeremie Vidal (ENS Lyon), Antoine Venaille (ENS Lyon). Through these collaborations, gaining a deep understanding of the physical models and questions raised by these researchers in their respective domains will be crucial. These domains include plasma physics for controlled nuclear fusion [5], geophysical wave currents [2], edge quantum states, and quantum molecular spectra, with the goal of establishing a solid mathematical framework.

The primary objective is to extend the model described in [4] to physical systems with boundaries and resonances (non normal operators)[6] and to illustrate the applications of these models in various domains of wave physics, including quantum waves, seismic waves, and plasma waves.

3 Requirements

The candidate is expected to have a strong background in theoretical physics and mathematical physics. Solid skills in scientific computing will be appreciated for illustrating the results.

References

[1] B. Booss and D.D. Bleecker. *Topology and analysis. The Atiyah-Singer index formula and gauge- theoretic physics. Transl. from the German by D. D. Bleecker and A. Mader.* Universitext. New York etc.: Springer-Verlag. XVI, 451, 1985.

[2] P. Delplace, J. B. Marston, and A. Venaille. Topological origin of equatorial waves. *Science*, 358(6366):1075–1077, 2017.

[3] Morteza Aghaee et al. Interferometric single-shot parity measurement in an inas-al hybrid device, 2024.

[4] F. Faure. Manifestation of the topological index formula in quantum waves and geophysical waves. *Annales Henri Lebesgue, link*, 6:449–492, 2023.

[5] Hong Qin and Yichen Fu. Topological langmuir-cyclotron wave, 2022.

[6] Furu Zhang and Jin-Han Xie. Non-hermitian topological wall modes in rotating rayleigh-benard convection, 2023.