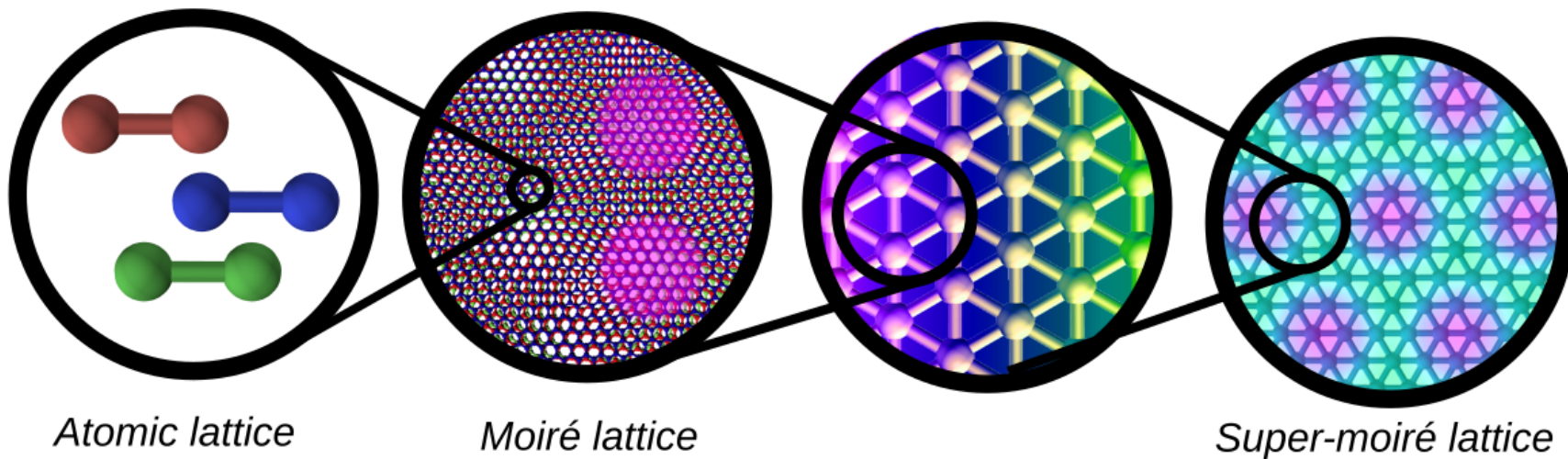


Solving topological and correlated super-moire materials with tensor networks and tensor cross interpolation

Jose Lado

Department of Applied Physics, Aalto University, Finland



2D Materials 12 (1), 015018 (2025)

arXiv:2503.04373 (2025)

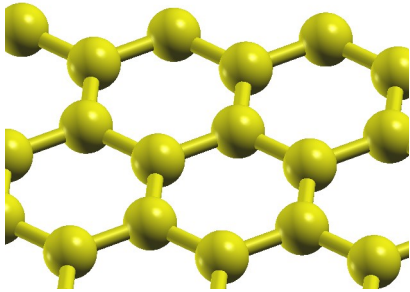
arXiv:2506.05230 (2025)

International workshop on tensor cross interpolation and other algorithms to learn tensor networks
Grenoble, France, October 6th 2025

The two-dimensional materials world

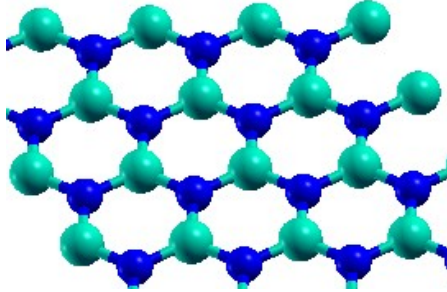
Semimetal

Graphene



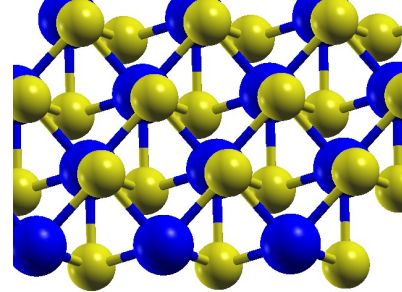
Insulator

BN



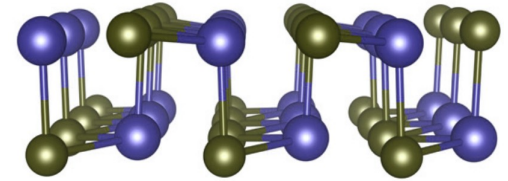
Superconductor

NbSe₂



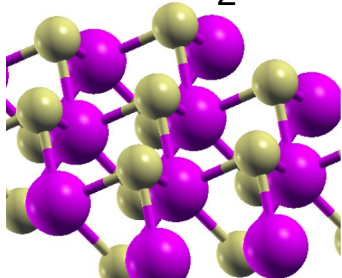
Ferroelectric

SnTe



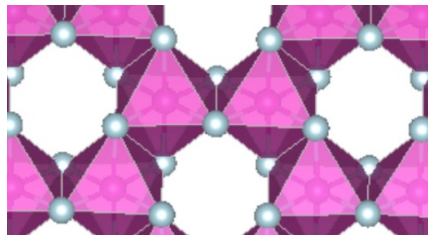
Semiconductor

WS₂



Ferromagnet

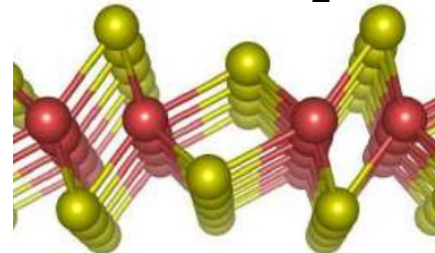
CrI₃



Quantum spin

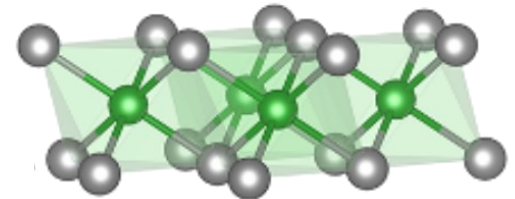
Hall insulator

WTe₂



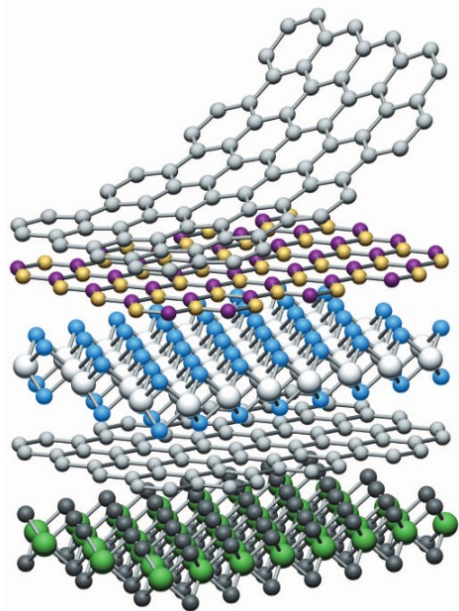
Multiferroic

NiI₂



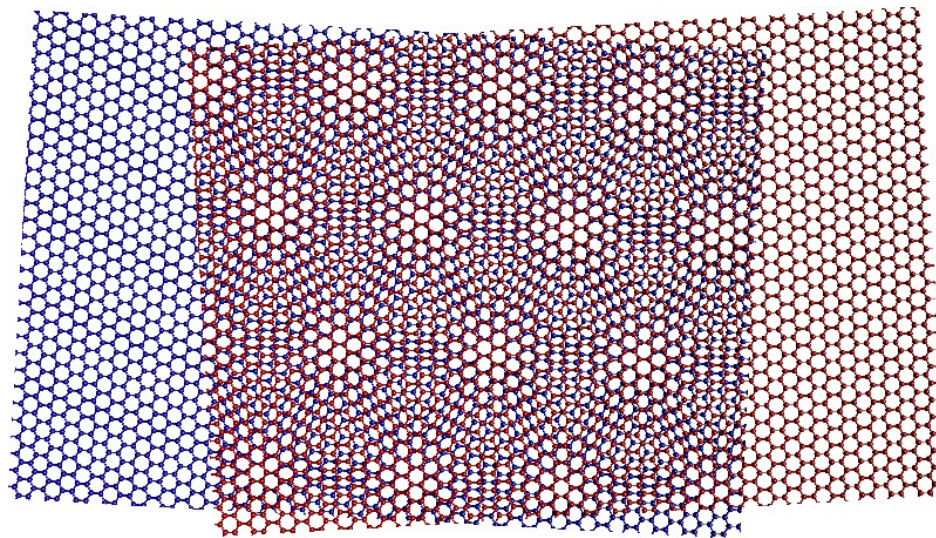
The flexibility of two-dimensional materials

They can be stacked



Nature 499, 419–425 (2013)

They can be rotated

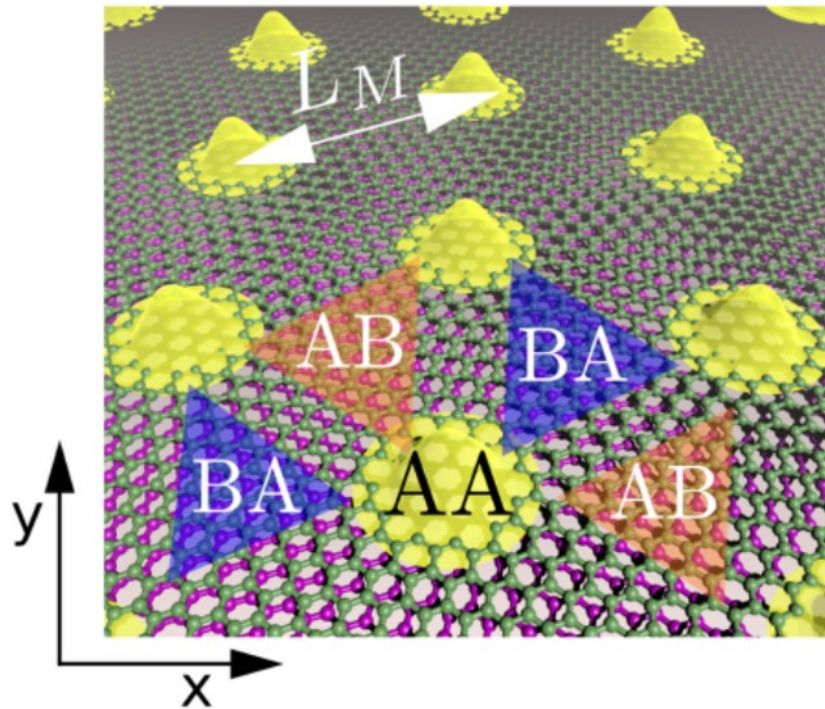


Science 361, 6403, 690-693 (2018)

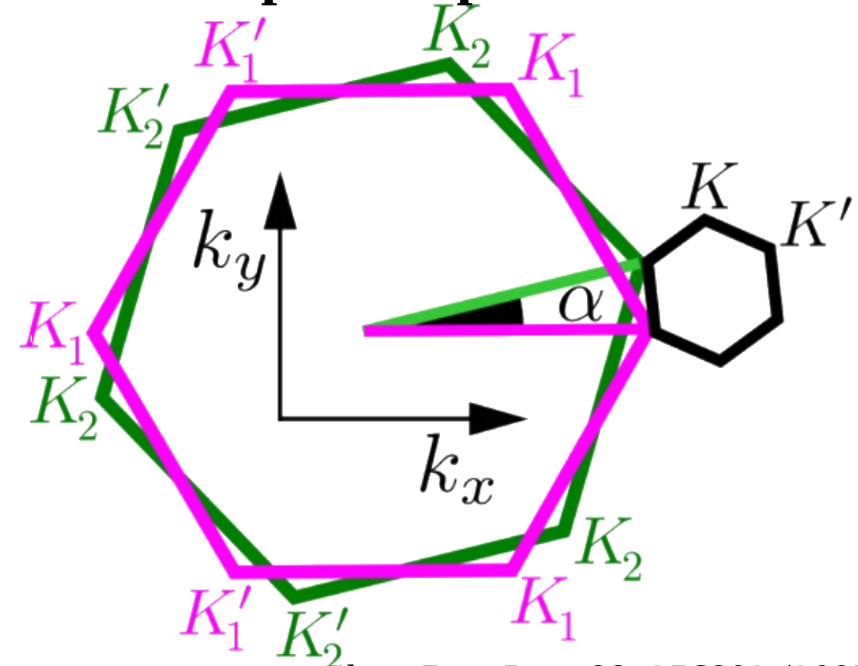
These are unique features of two-dimensional materials

Twisted graphene bilayers

Structure



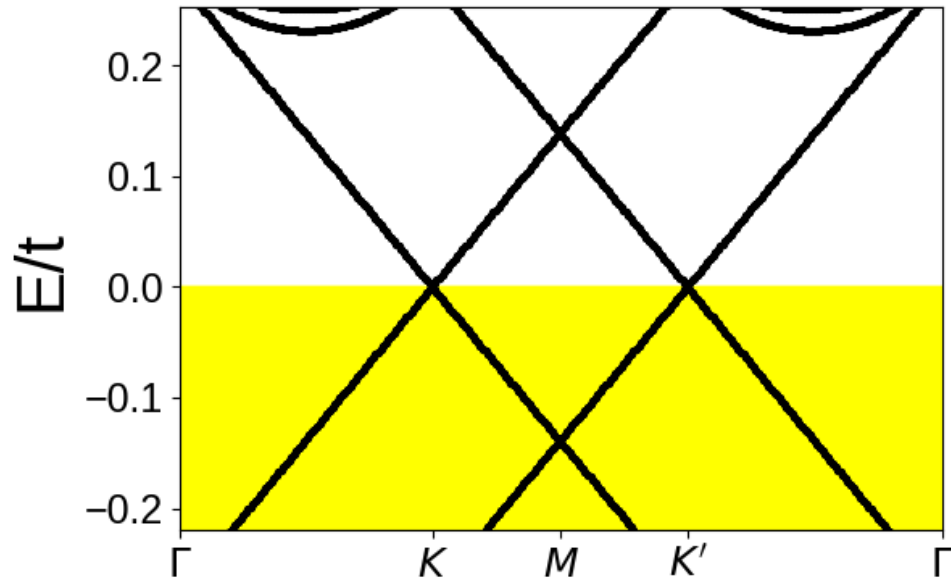
Reciprocal space



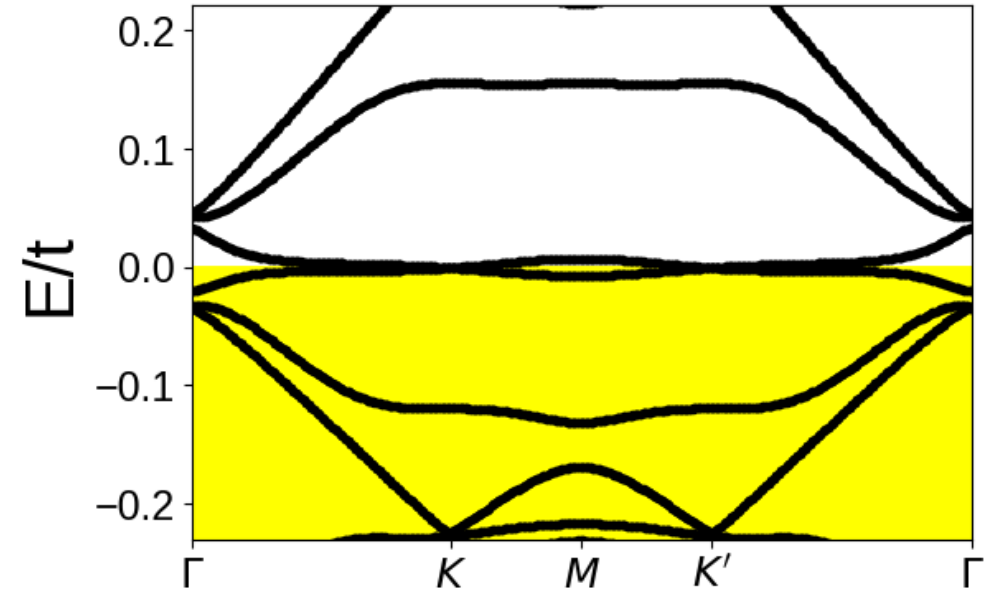
Phys. Rev. Lett. 99, 256802 (2007)
Phys. Rev. B 82, 121407(R) (2010)
PNAS 108 (30) 12233-12237 (2011)

Electronic structure of twisted bilayer

Uncoupled layers



Coupled layers

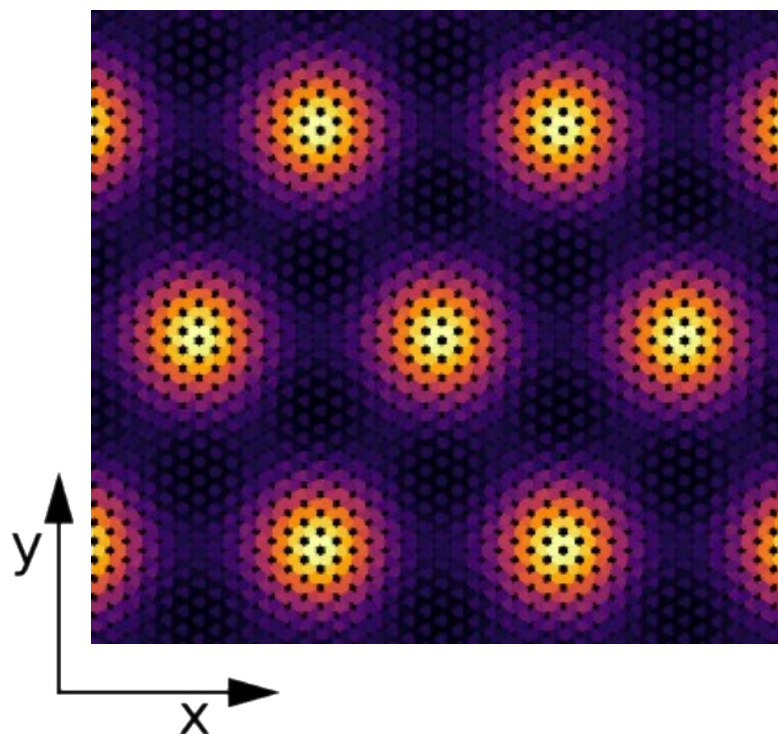


Twisted graphene bilayers feature flat bands driven by the twist

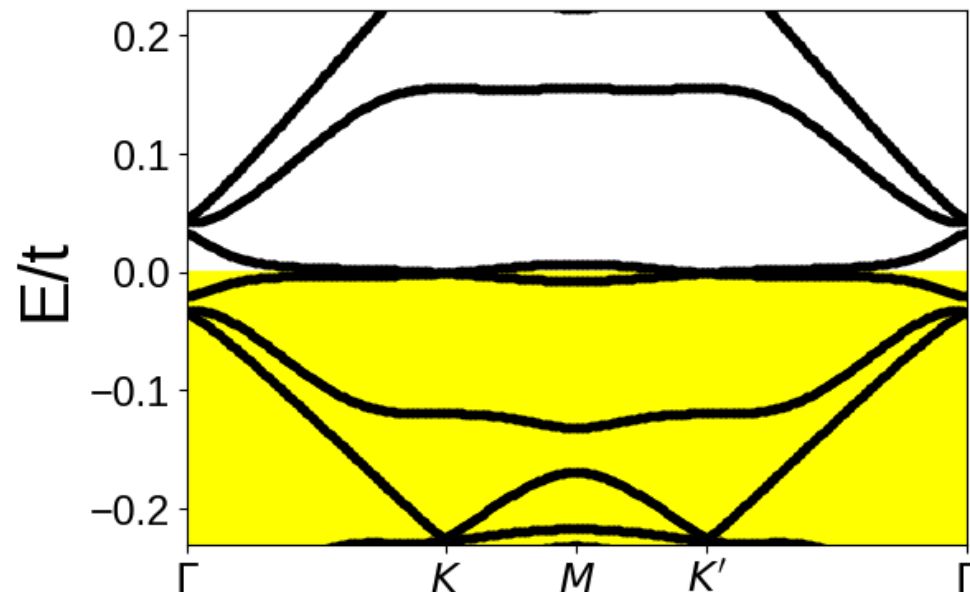
Phys. Rev. Lett. 99, 256802 (2007)
Phys. Rev. B 82, 121407(R) (2010)
PNAS 108 (30) 12233-12237 (2011)

Electronic structure of twisted bilayer

Local density of states



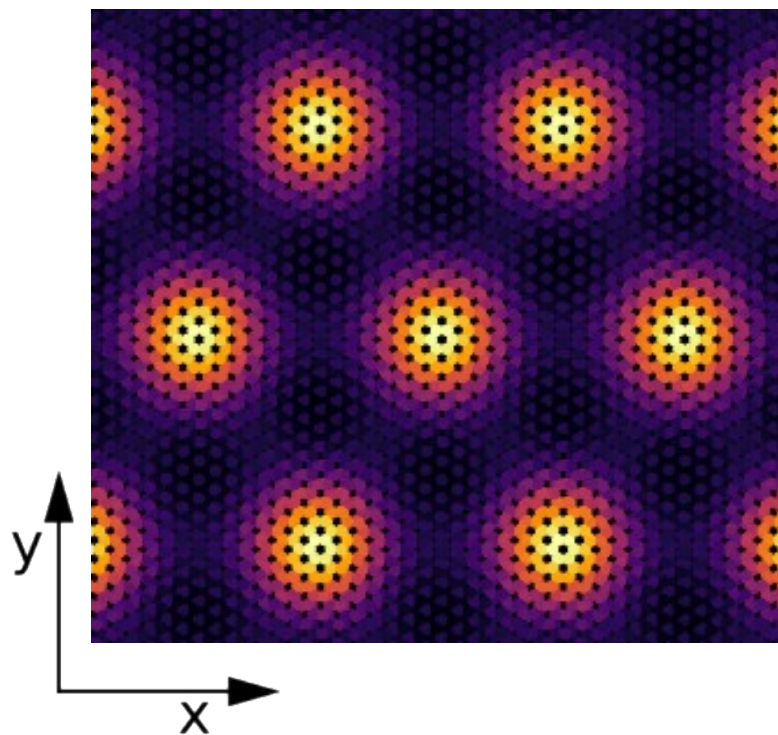
Coupled layers



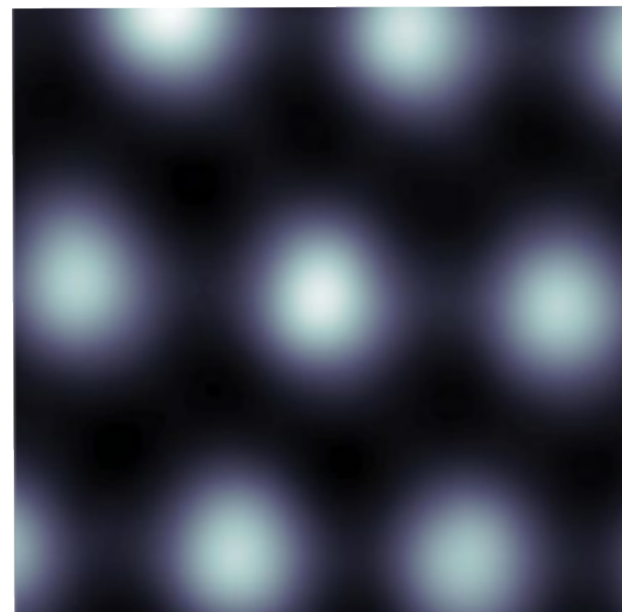
Phys. Rev. Lett. 99, 256802 (2007)
Phys. Rev. B 82, 121407(R) (2010)
PNAS 108 (30) 12233-12237 (2011)

Electronic structure of twisted bilayer

Local density of states



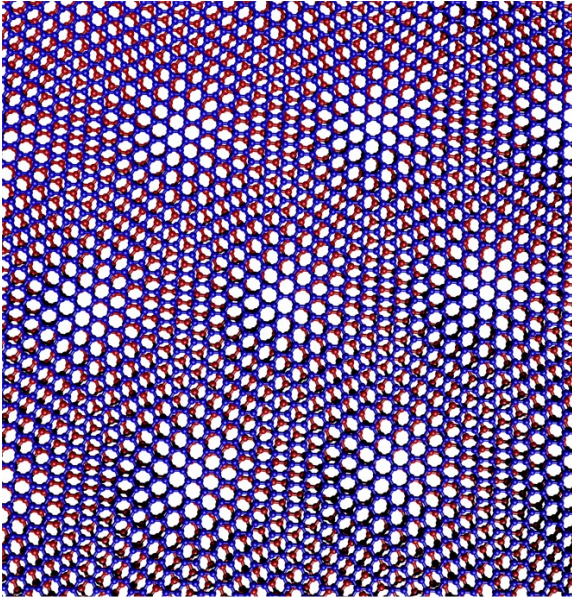
Experimental local density of states



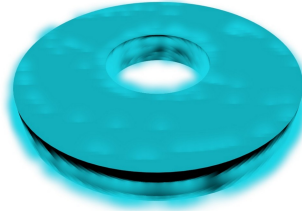
Nature Physics 6, 109–113 (2010)
Nature 572, 101–105 (2019)
Nature 572, 95–100 (2019)

The tunability of twisted van der Waals materials

Twisted bilayer graphene

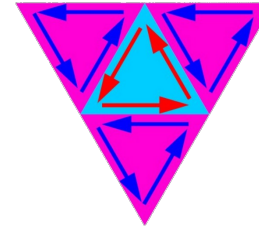


Superconductivity



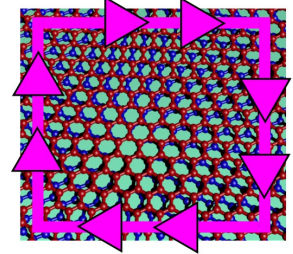
Nature 556, 43–50 (2018)

Topological networks



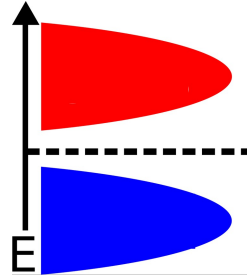
Nano Lett. 18, 11, 6725-6730 (2018)

Chern insulators



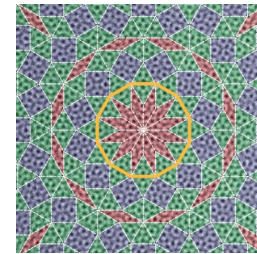
Science 365, 605-608 (2019)

Correlated insulators



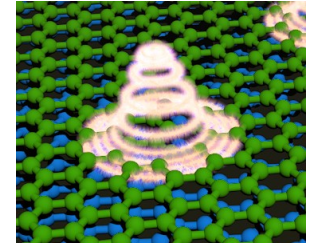
Nature 556, 80–84 (2018)

Quasicrystalline physics



Science 361, 782-786 (2018)

Proximal fractional Chern insulators

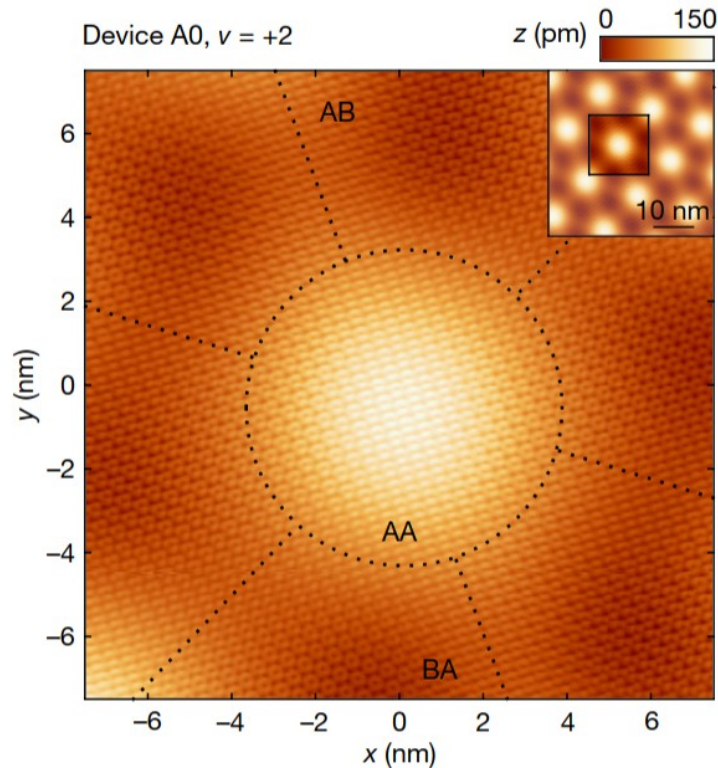


Nature 600, 439–443 (2021)

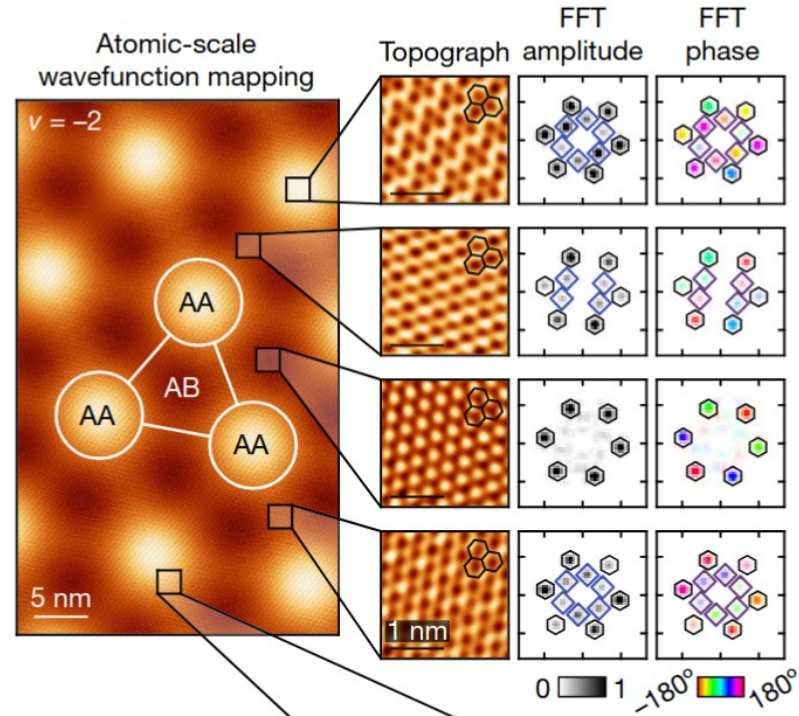
Twisted multilayers provide a powerful platform for emergent phenomena

Experimental observation of multiple scales in twisted bilayers

Moire and atomic scale with STM

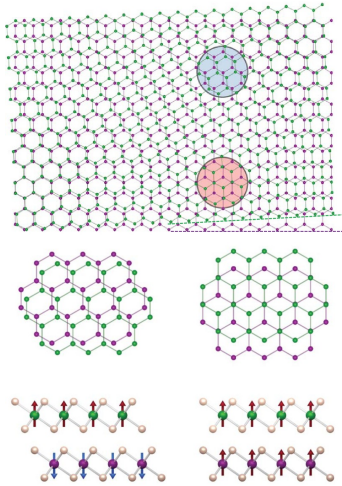


Atomic-scale symmetry breaking with STM



Artificial quantum matter in moire van der Waals heterostructures

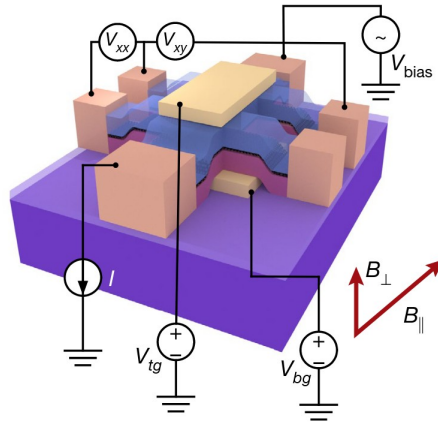
Unconventional magnets



Twisted CrBr_3

Science, 374(6571),
1140–1144 (2021)

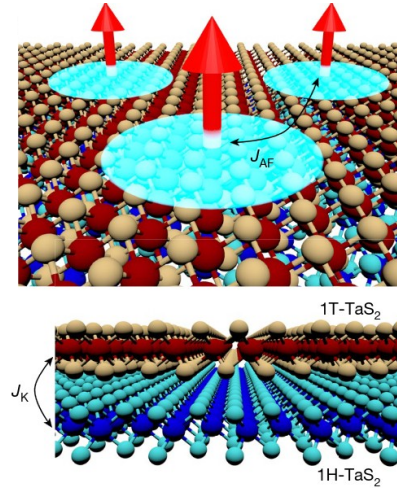
Unconventional superconductors



Twisted trilayer
graphene

Nature 595,
526–531 (2021)

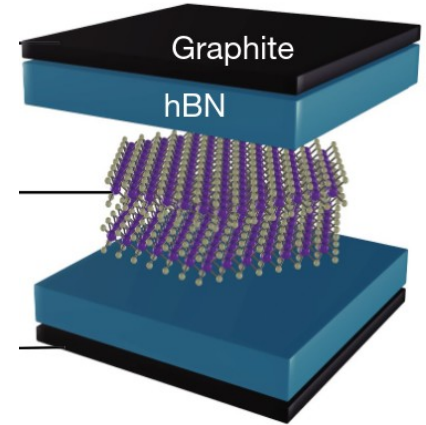
Heavy-fermion quantum materials



1H-1T TaS_2

Nature 599,
582–586 (2021)

Fractional topological matter

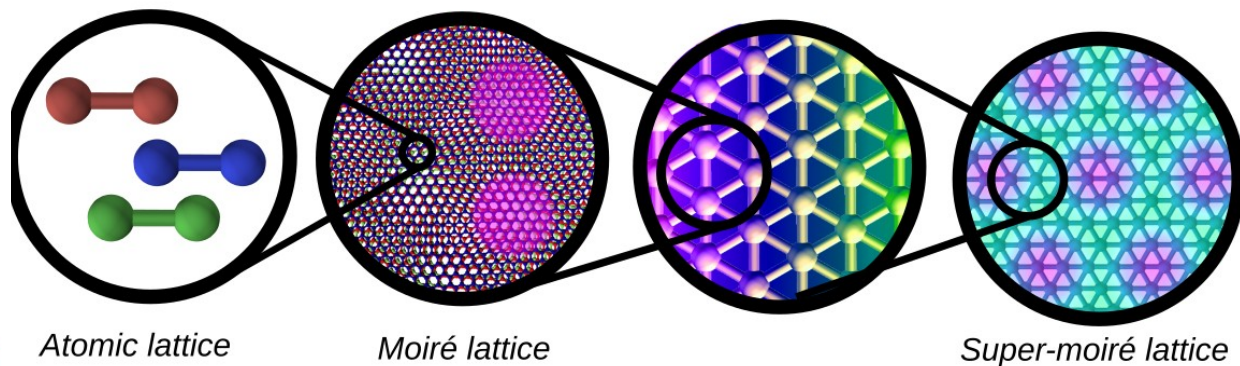
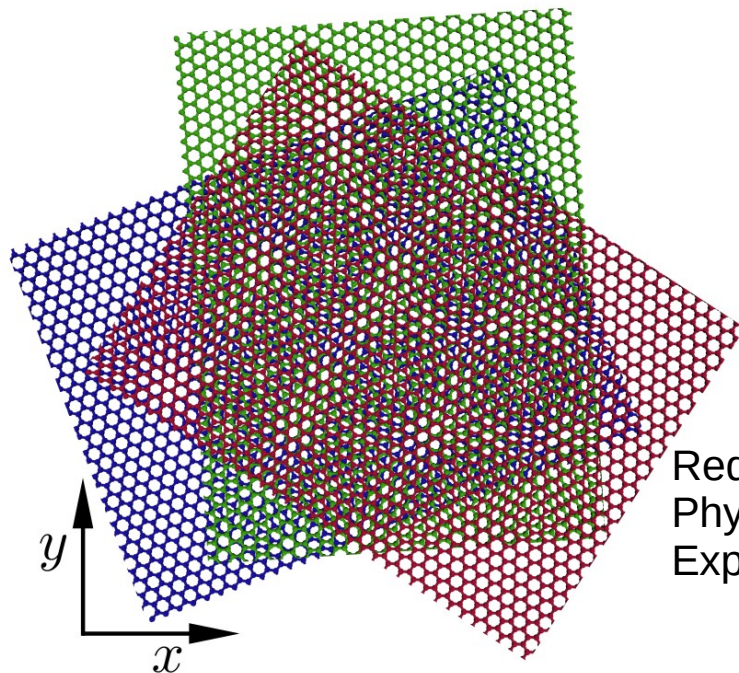


Twisted MoTe_2

Nature 622,
63–68 (2023)

Super-moire materials

Moire-of-moire materials



Requires computing up to a billion sites (10^9)
Physics at several length scales (atomic, moire, and super-moire)
Experiments showing correlations and unconventional superconductivity

Nature 620, 762–767 (2023)

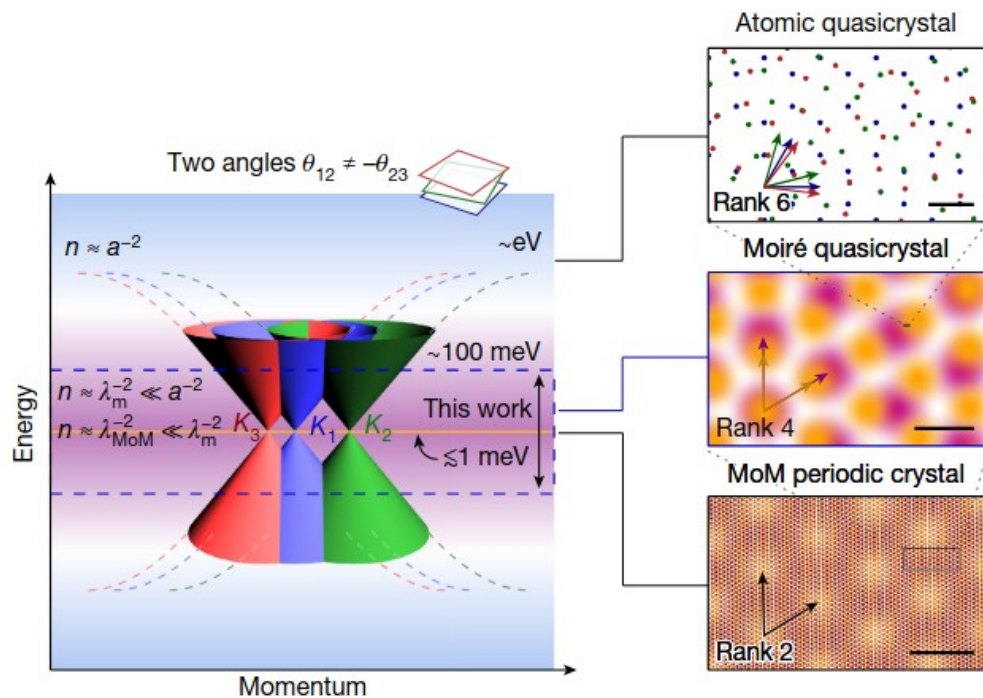
Nature 625, 494–499 (2024)

Nature 641, 896–903 (2025)

How can we compute the electronic structure of exceptionally large systems?

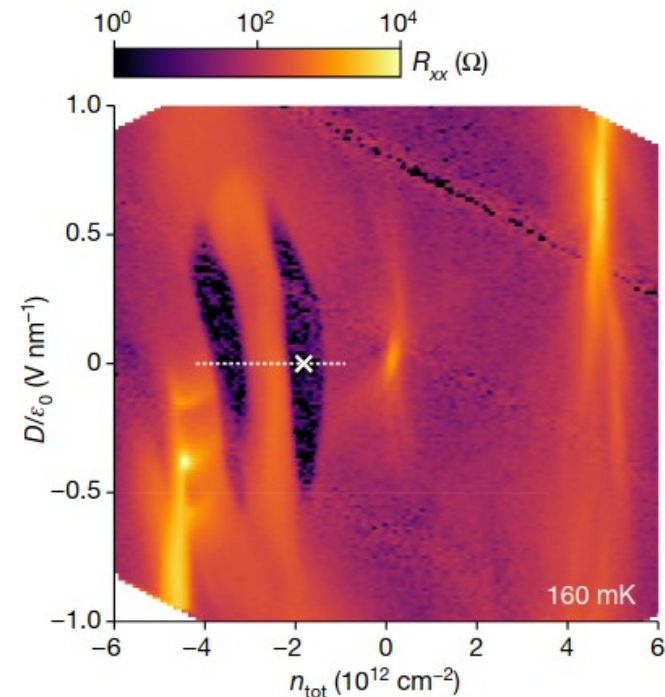
Super-moire materials, experimentally

Moire quasicrystal in twisted trilayer graphene

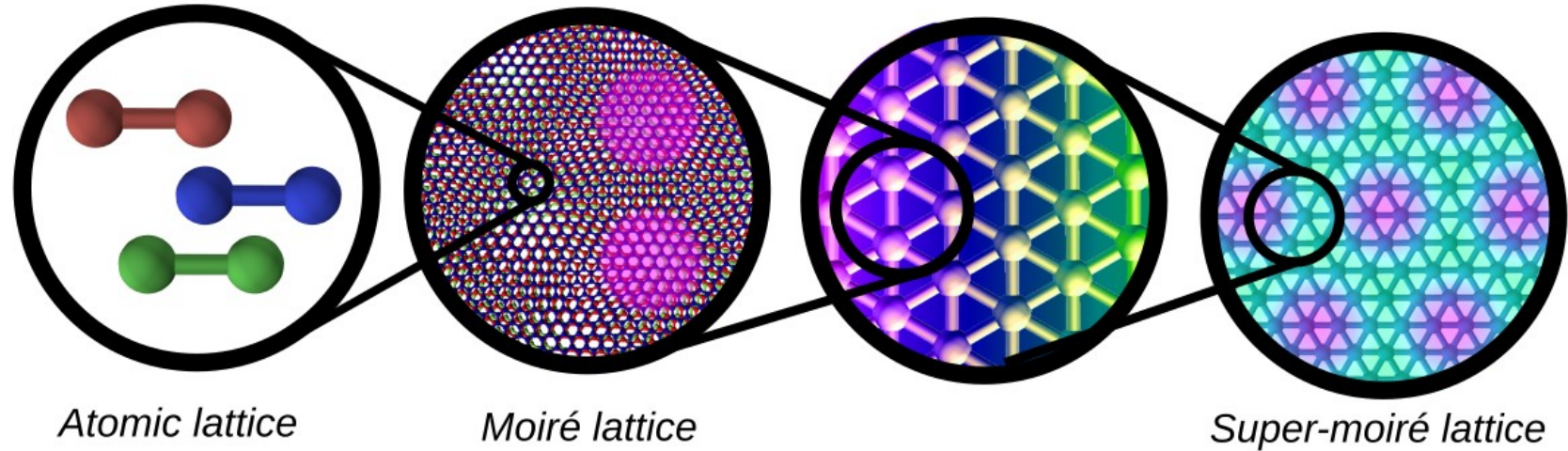


Nature 620, 762–767 (2023)

Coexisting correlations and superconductivity



Why super-moire materials are hard (for theorists)



Moiré systems **$N=10^5$ sites**, calculations taking around 1 minute

Computational cost grows as **N** (best case) or **N^3** (worst case)

Super-moiré **$N=10^9$ sites**, calculations taking 10 days – 1000000 years

Is there a way to solve exceptionally large electronic structure problems?

(even going beyond usual memory limitations of conventional methods)

Behind the scenes

Yitao Sun



Tiago Antão



Marcel Niedermeier



Adolfo Fumega



Correlated states in super-moiré materials with a kernel polynomial quantics tensor cross interpolation algorithm, AO Fumega, M Niedermeier, JL Lado, **2D Materials 12 (1), 015018 (2025)**

Self-consistent tensor network method for correlated super-moire matter beyond one billion sites, Y Sun, M Niedermeier, TVC Antão, AO Fumega, JL Lado, **arXiv:2503.04373 (2025)**

Tensor network method for real-space topology in quasicrystal Chern mosaics, TVC Antão, Y Sun, AO Fumega, JL Lado, **arXiv:2506.05230 (2025)**

Tensor networks for interacting super-moire materials

Modeling electrons in a material

Define operators that can create or destroy particles

c_i Annihilation operator, destroys a particle in site i

$c_i^\dagger$ Creation operator, creates a particle in site i

The empty vacuum state $|\Omega\rangle$ is defined as $c_i |\Omega\rangle = 0$

The Hamiltonian is written in terms of creation and annihilation operators

$$H = c_0^\dagger c_1 + h.c.$$

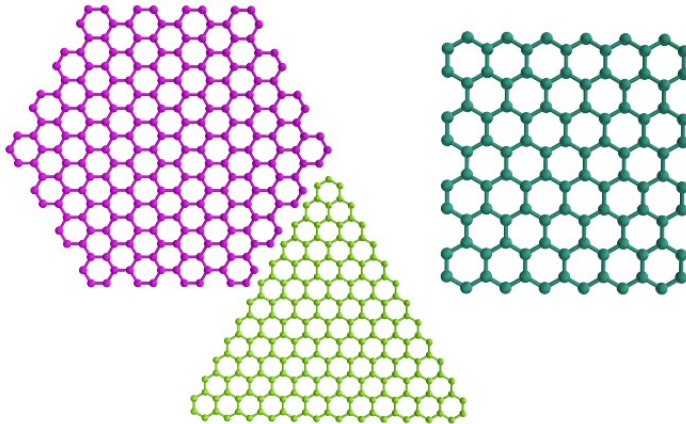
Modes in a single particle Hamiltonian

To solve a single particle Hamiltonian, we just have to diagonalize the matrix

$$H = \sum_{ij} t_{ij} c_i^\dagger c_j$$

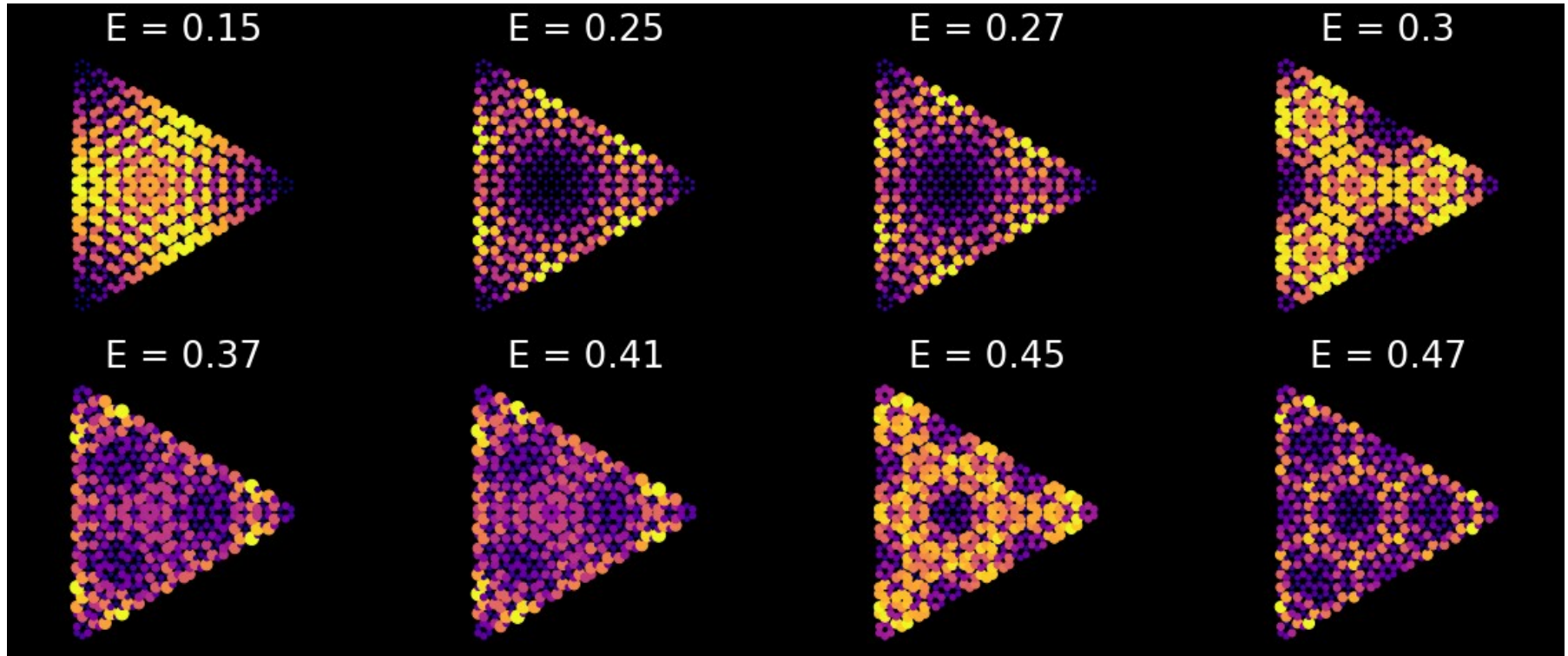
$$H = \sum_{\alpha} \epsilon_{\alpha} \Psi_{\alpha}^{\dagger} \Psi_{\alpha}$$

If take a certain finite geometry, we can find the confined modes

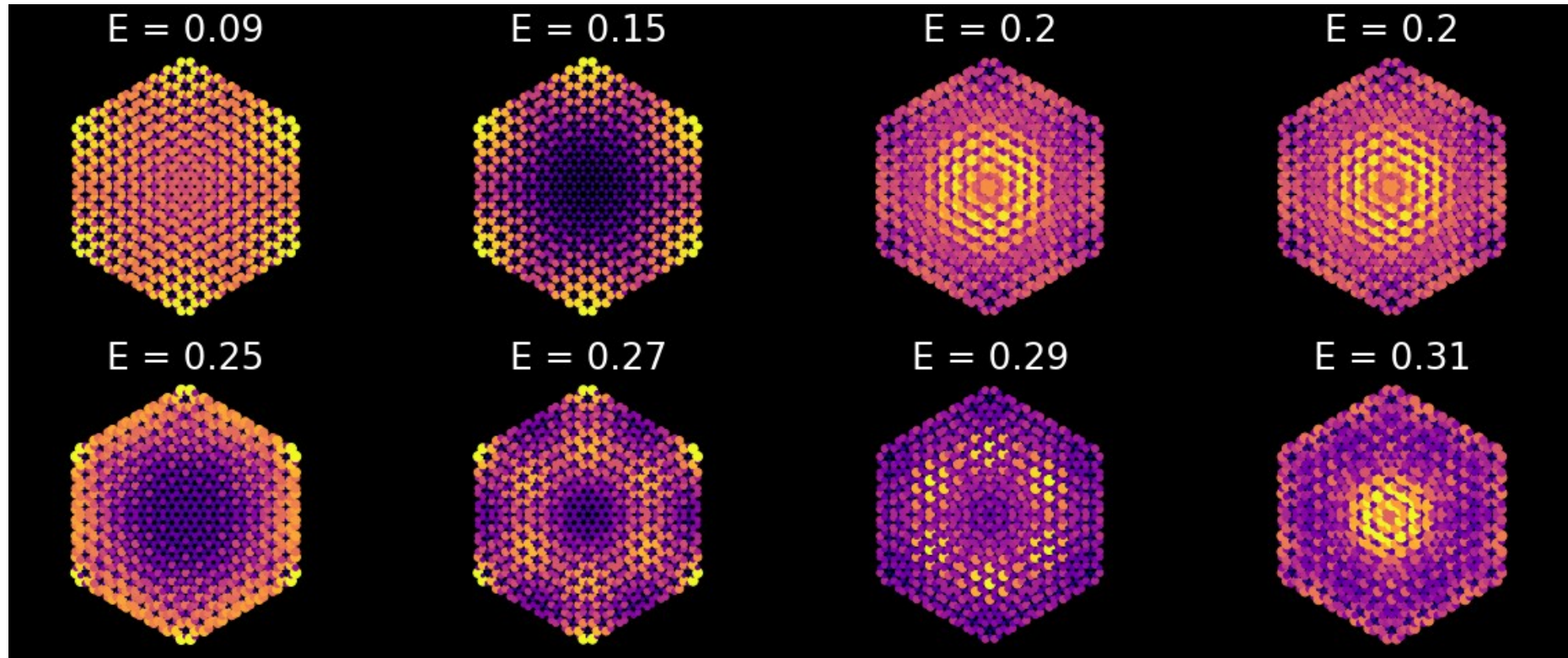


Let us do it for graphene islands

Confined modes in graphene islands



Confined modes in graphene islands



The challenge of correlated super-moire materials

Hamiltonian describing electrons in a super-moire material

$$H = H_0 + H_V = \sum_{\alpha\beta} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \sum_{\alpha\beta} V_{\alpha\beta} c_{\alpha}^{\dagger} c_{\alpha} c_{\beta}^{\dagger} c_{\beta}$$

Mean-field treatment of the interacting Hamiltonian

$$H^{MF} = \sum_{\alpha\beta} (t_{\alpha\beta} + \chi_{\alpha\beta}) c_{\alpha}^{\dagger} c_{\beta} = \sum_{\alpha\beta} H_{\alpha\beta}^{MF} c_{\alpha}^{\dagger} c_{\beta}$$

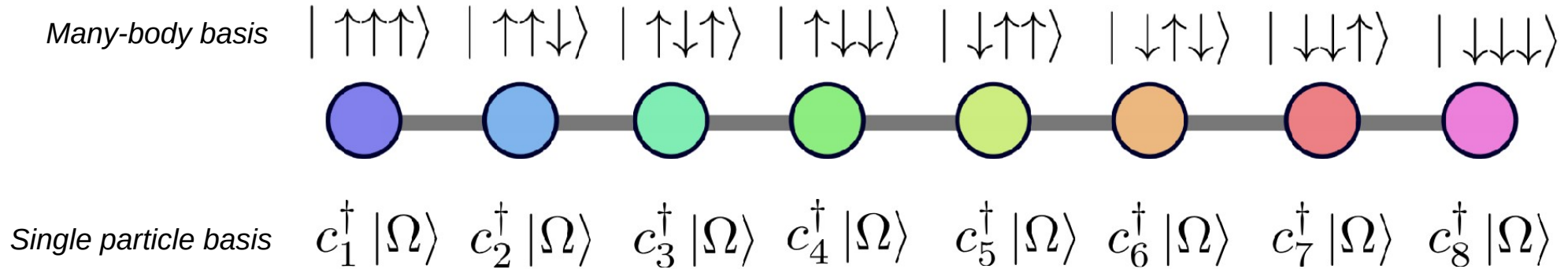
Solving the system requires dealing with matrices proportional to the system size

In a super-moire system, this requires solving a billion sites

How could we solve a system whose Hamiltonian would be too large to store?
(even before considering the time required to solve it)

Tensor network for single particle tight binding problems

We can identify a many-body space with a very large single particle one



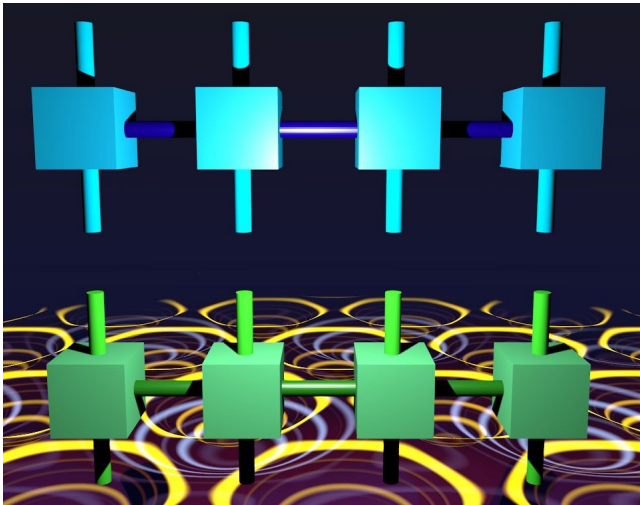
We can use tensor networks to solve very large tight binding problem

Dealing with very large spaces with tensor networks

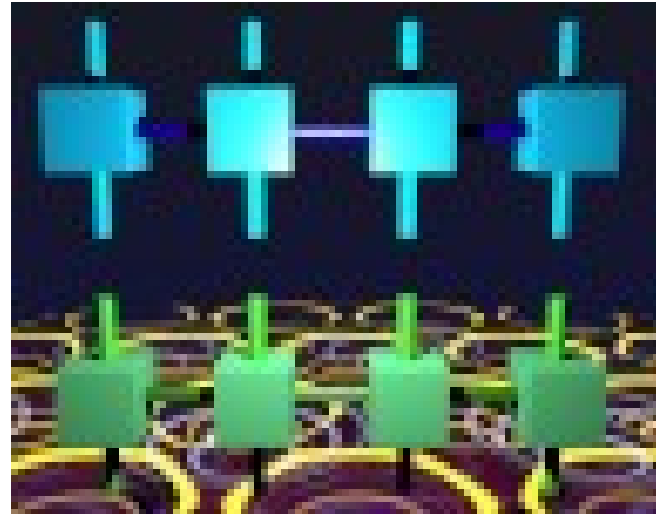
A many-body wavefunction is a very high dimensional object (2^L coefficients)

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

Tensor-networks allow “compressing” exponentially large information with linear resources



“True wavefunction”



“Tensor-network wavefunction”

Dealing with very large spaces with tensor networks

Typical many-body wavefunction $|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$

We need to determine in total 2^L coefficients

Is there an efficient way of storing so many coefficients?

Tensor networks allow parametrizing many-body wavefunctions as

$$|\Psi\rangle = \sum_{\{s\}} \text{Tr} \left[M_1^{(s_1)} M_2^{(s_2)} \dots M_N^{(s_N)} \right] |s_1 s_2 \dots s_N\rangle$$

$L\chi^2$ parameters



Tensor networks allow to drastically reduce the memory required to store a many-body state

Exponentially large algebra with tensor networks

Tensor network allow to (approximately) operate in exponentially large vector spaces

vector

$$|\Psi\rangle \equiv \text{[Diagram: A horizontal chain of 5 cyan squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center.]}$$

matrix

$$\hat{A} \equiv \text{[Diagram: A horizontal chain of 5 red squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

$$|\Psi_2\rangle = \hat{A}|\Psi_1\rangle \equiv \text{[Diagram: A 2x5 grid of squares. The top row consists of 5 red squares and the bottom row consists of 5 cyan squares. Horizontal lines connect squares in each row. Vertical lines extend upwards from the top row and downwards from the bottom row.]} = \text{[Diagram: A horizontal chain of 5 blue squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

$$\hat{A}_3 = \hat{A}_1 + \hat{A}_2 \equiv \text{[Diagram: A 2x5 grid of squares. The top row consists of 5 red squares and the bottom row consists of 5 orange squares. Horizontal lines connect squares in each row. Vertical lines extend upwards from the top row and downwards from the bottom row.]} + \text{[Diagram: A horizontal chain of 5 orange squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]} = \text{[Diagram: A horizontal chain of 5 teal squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

Learning the tensor network representation

We need to represent all the tight binding terms as tensor networks

- In some cases, we can build the initial tensor network explicitly
- In other cases, we learn the tensor network with quantum tensor cross interpolation

$$\left(\begin{array}{cccccccccccccccc} \bullet & \bullet & \circ & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right) \approx \left(\begin{array}{ccc} \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array} \right) \left(\begin{array}{ccc} \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array} \right)^{-1} \left(\begin{array}{cccccccccccccccc} \bullet & \bullet & \circ & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Phys. Rev. Lett. 132, 056501 (2024)

SciPost Phys. 18, 104 (2025)

Self-consistent electronic interactions with tensor networks

Super-moire interacting Hamiltonian

$$H = H_0 + H_V = \sum_{\alpha\beta} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \sum_{\alpha\beta} V_{\alpha\beta} c_{\alpha}^{\dagger} c_{\alpha} c_{\beta}^{\dagger} c_{\beta}$$

Mean-field decoupled Hamiltonian

$$H^{MF} = \sum_{\alpha\beta} (t_{\alpha\beta} + \chi_{\alpha\beta}) c_{\alpha}^{\dagger} c_{\beta} = \sum_{\alpha\beta} H_{\alpha\beta}^{MF} c_{\alpha}^{\dagger} c_{\beta} \quad \mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[Diagram: A chain of orange squares connected by horizontal lines, representing the mean-field decoupled Hamiltonian tensor network.]}$$

Tensor network representation of the correlators

$$\langle c_{\alpha}^{\dagger} c_{\beta} \rangle = \langle \alpha | \Xi(\mathcal{H}^{MF}) | \beta \rangle \equiv \text{[Diagram: A chain of cyan squares connected by horizontal lines, representing the tensor network representation of the correlator.]}$$

Self-consistent electronic interactions with tensor networks

We represent the super-moire electronic Hamiltonian as a tensor-network

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \Gamma_{s_1, s'_1}^{(1)} \Gamma_{s_2, s'_2}^{(2)} \Gamma_{s_3, s'_3}^{(3)} \dots \Gamma_{s_L, s'_L}^{(L)} \quad \mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]}$$

The mean-field problem can be reformulated purely with tensor networks

Tensor Network SCF loop

$$\begin{aligned} \mathcal{H}_{\alpha\beta}^{MF} &\equiv \text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]} = t_{\alpha\beta} + \chi_{\alpha\beta} \left(\text{[Diagram: A sequence of cyan squares connected horizontally, representing a tensor network.]} \right) \\ \langle c_{\alpha}^{\dagger} c_{\beta} \rangle &\equiv \text{[Diagram: A sequence of cyan squares connected horizontally, representing a tensor network.]} = \sum_n \lambda_n T_n \left(\text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]} \right) \end{aligned}$$

(Note: Dashed arrows in the original image indicate a self-consistent loop between the two equations.)

Spectral functions with the kernel polynomial method

Let us expand a delta function in Chebyshev polynomials

$$\delta(\omega - \mathcal{H}) = \sum_n \lambda_n(\omega) T_n(\mathcal{H})$$

Chebyshev relation

$$T_n(\mathcal{H}) = 2\mathcal{H}T_{n-1}(\mathcal{H}) - T_{n-2}(\mathcal{H})$$

Expansion coefficients for delta function

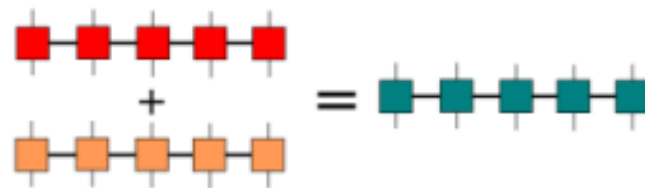
$$\lambda_n(\omega) = \frac{2T_n(\omega)}{\pi\sqrt{1-\omega^2}}$$

Rev. Mod. Phys. 78, 275 (2006)

KPM can be implemented with tensor networks by using tensor network algebra

Phys. Rev. B 83, 195115 (2011)

Phys. Rev. Research 1, 033009 (2019)



Spectral functions with tensor networks

Local spectral function of the mean-field Hamiltonian

$$D(\omega, \alpha) = \langle \alpha | \left[\sum_n T_n(\mathcal{H}^{MF}) P_n(\omega) \right] | \alpha \rangle$$

With a tensor network Chebyshev algorithm

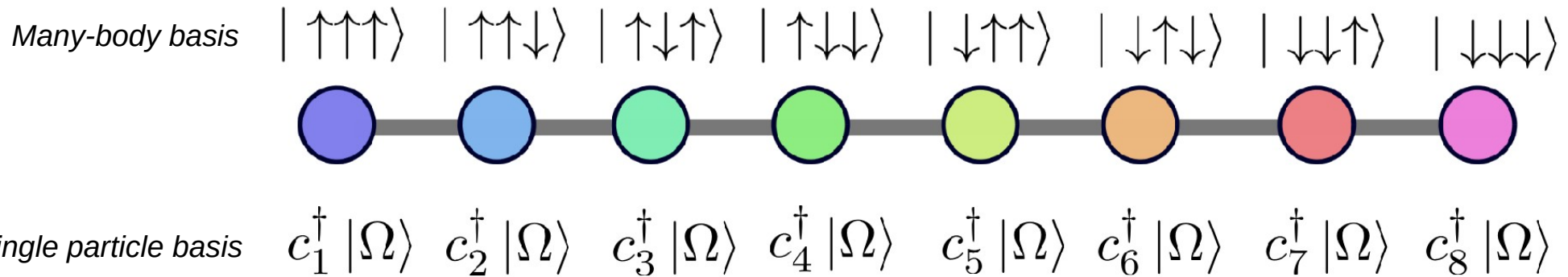
$$P_n(\omega) = \frac{2T_n(\omega)}{\pi\sqrt{1-\omega^2}}$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \begin{array}{c} \text{---} \square \text{---} \square \text{---} \dots \text{---} \square \text{---} \\ | \quad | \quad \quad \quad | \quad \quad \quad | \quad \quad \quad | \\ | \quad | \quad \quad \quad | \quad \quad \quad | \quad \quad \quad | \end{array}$$

Tensor network representation of a super-moire Hamiltonian

We can identify a many-body space with a very large single particle one



In the tight binding basis uniform hopping takes the form

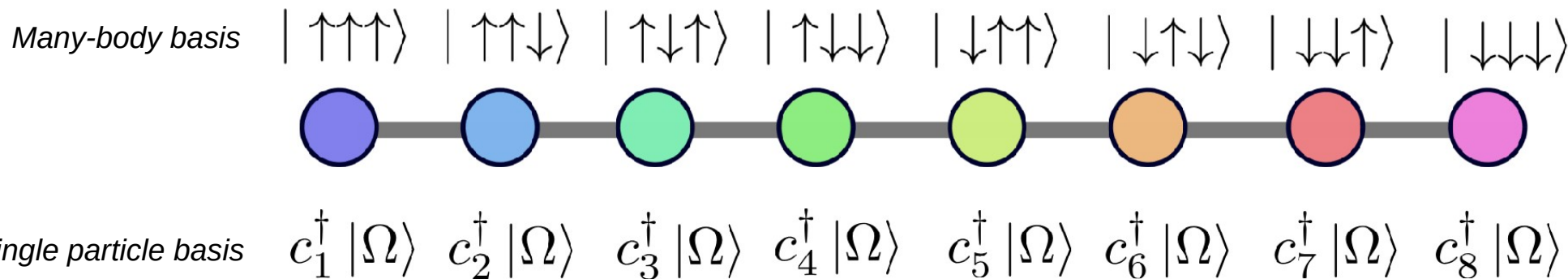
$$H_{0,NN} = \sum_{\alpha,s}^{N-1} t(c_{x_{\alpha+1},s}^{\dagger} c_{x_{\alpha},s} + h.c.)$$

In the tensor-network pseudospin basis, uniform hopping takes the form

$$\mathcal{H}_{0,NN} = \sum_{l,s}^L t(\sigma_{l,s}^{+} \otimes_{m>l} \sigma_{m,s}^{-} + h.c.)$$

Tensor network representation of a super-moire Hamiltonian

We can identify a many-body space with a very large single particle one



The tensor-network moire hopping can be built as

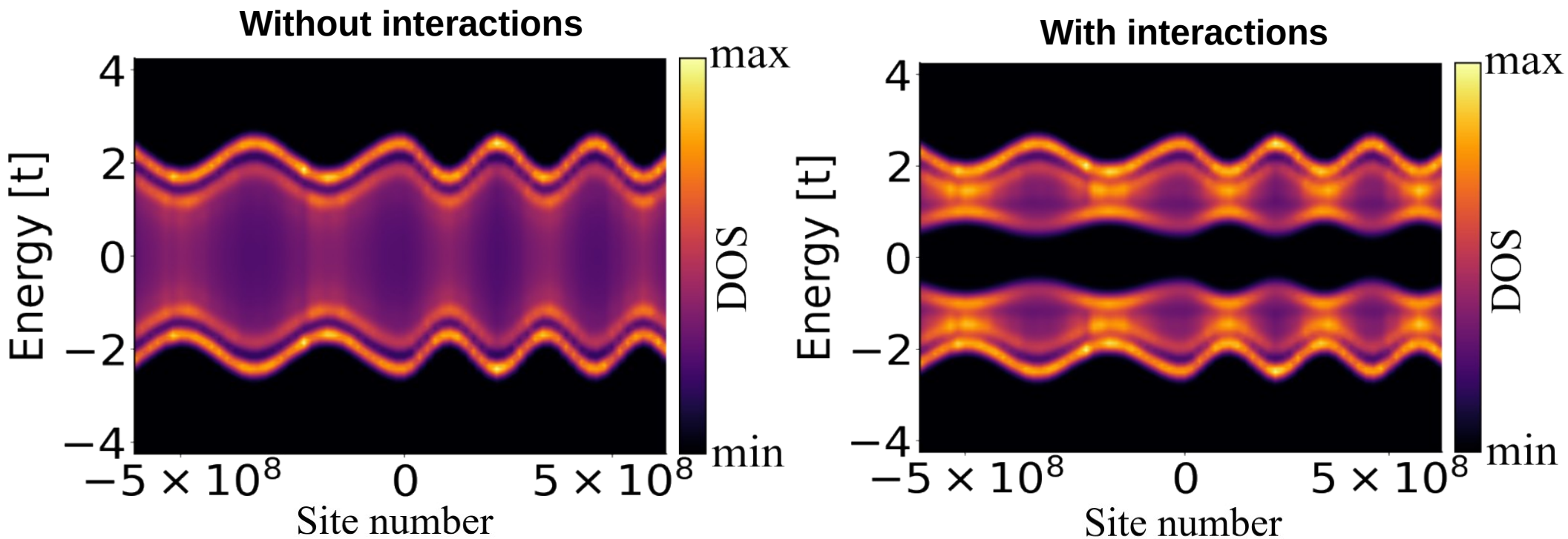
Find the MPS representation for the modulation and store in a diagonal MPO $\mathcal{T}$

$$|\tau\rangle = \sum M_{s_1}^{(1)} M_{s_2}^{(2)} M_{s_3}^{(3)} \dots M_{s_L}^{(L)} |s_1, s_2, \dots, s_L\rangle \quad \text{Quantics tensor cross interpolation}$$

Modulated super-moire by construction

$$\mathcal{H}_0 = \{[\mathcal{T} \sum_{l,s}^L (\sigma_{l,s}^+ \otimes_{m>l} \sigma_{m,s}^-)] + h.c.\}$$

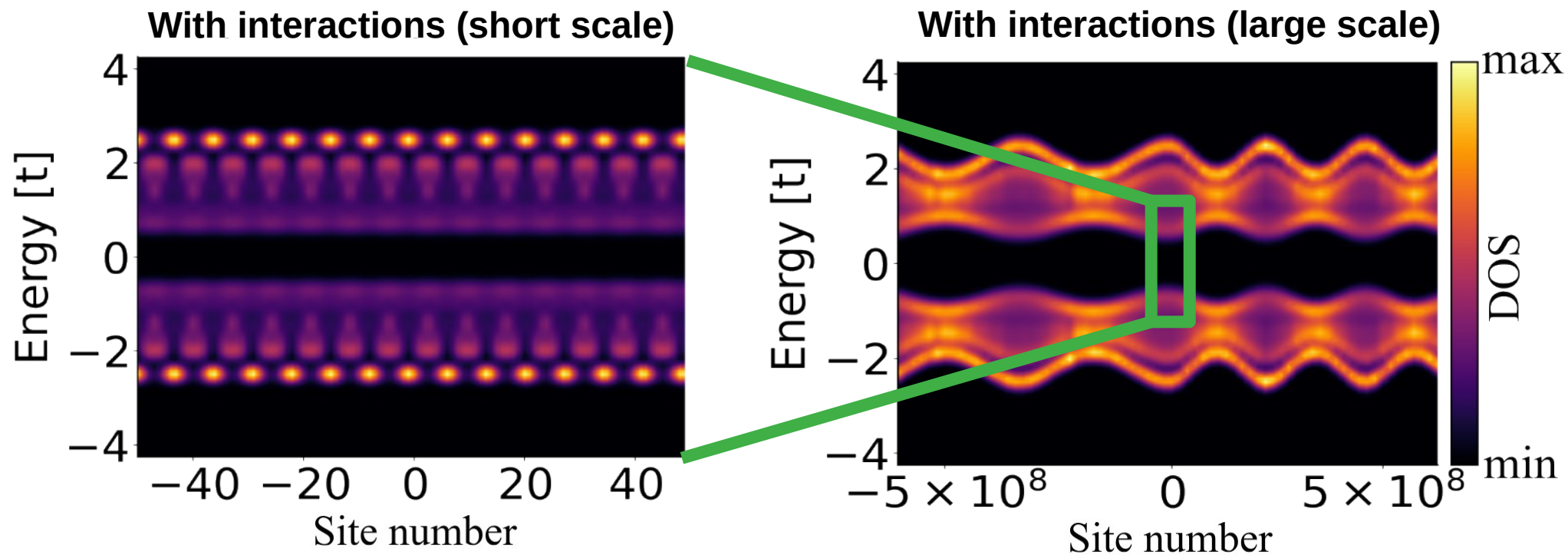
Solving billion-size super-moire materials



Tensor networks allow to solve selfconsistently a super-moire with one billion sites

arXiv:2503.04373 (2025)

Solving billion-size super-moire materials

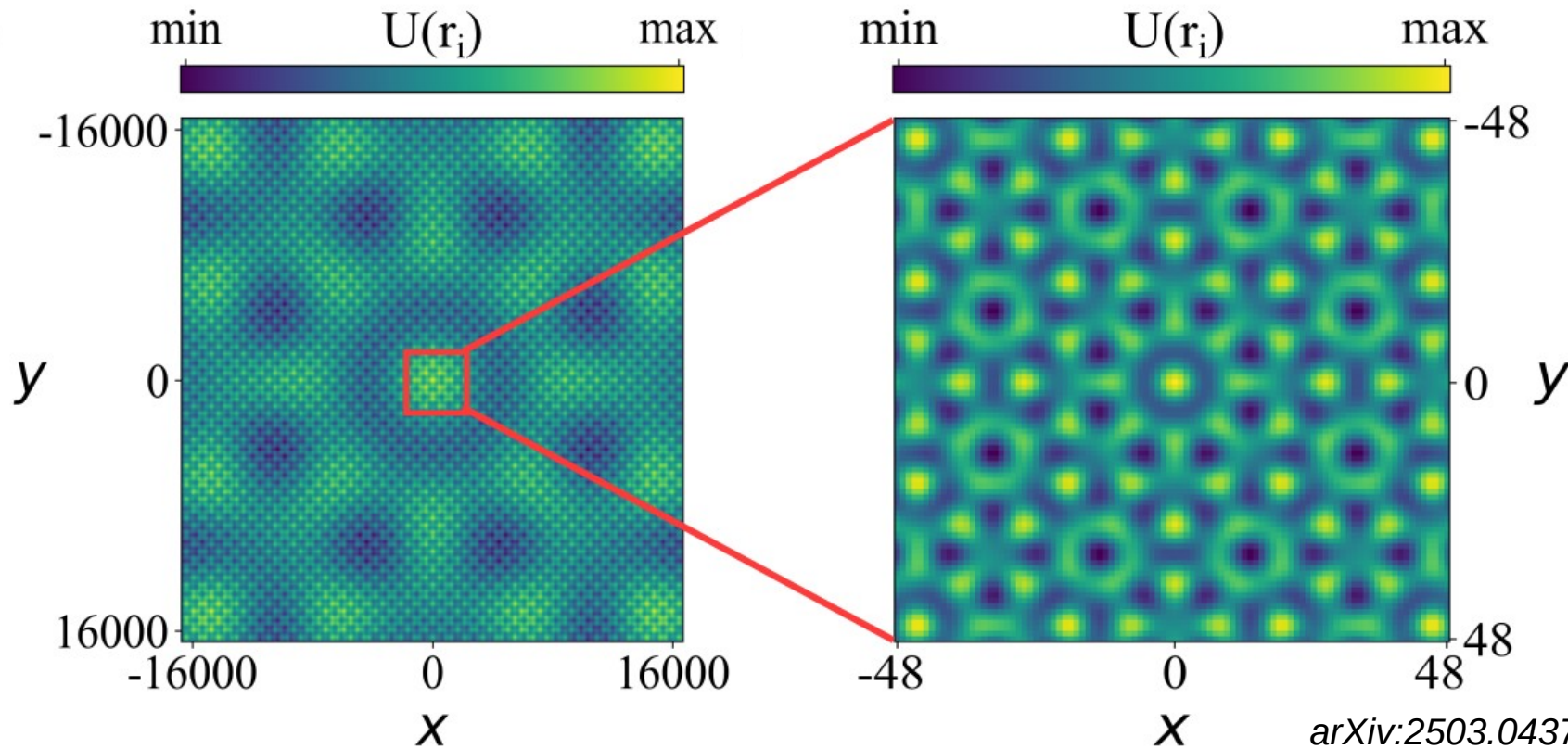


Tensor networks allow to solve selfconsistently a super-moire with one billion sites

arXiv:2503.04373 (2025)

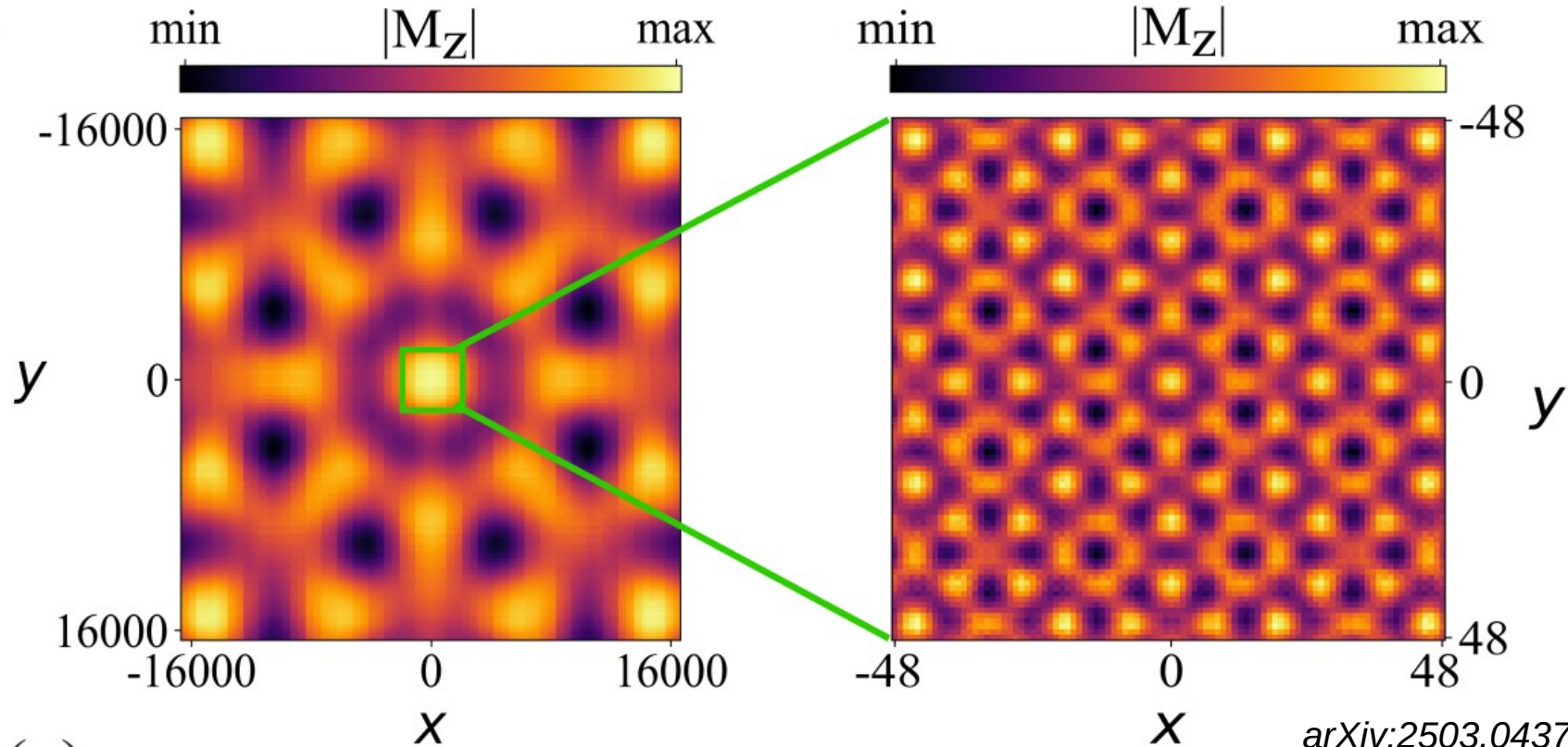
Two-dimensional billion size interacting super-moire

Modulation fo the Hamiltonian



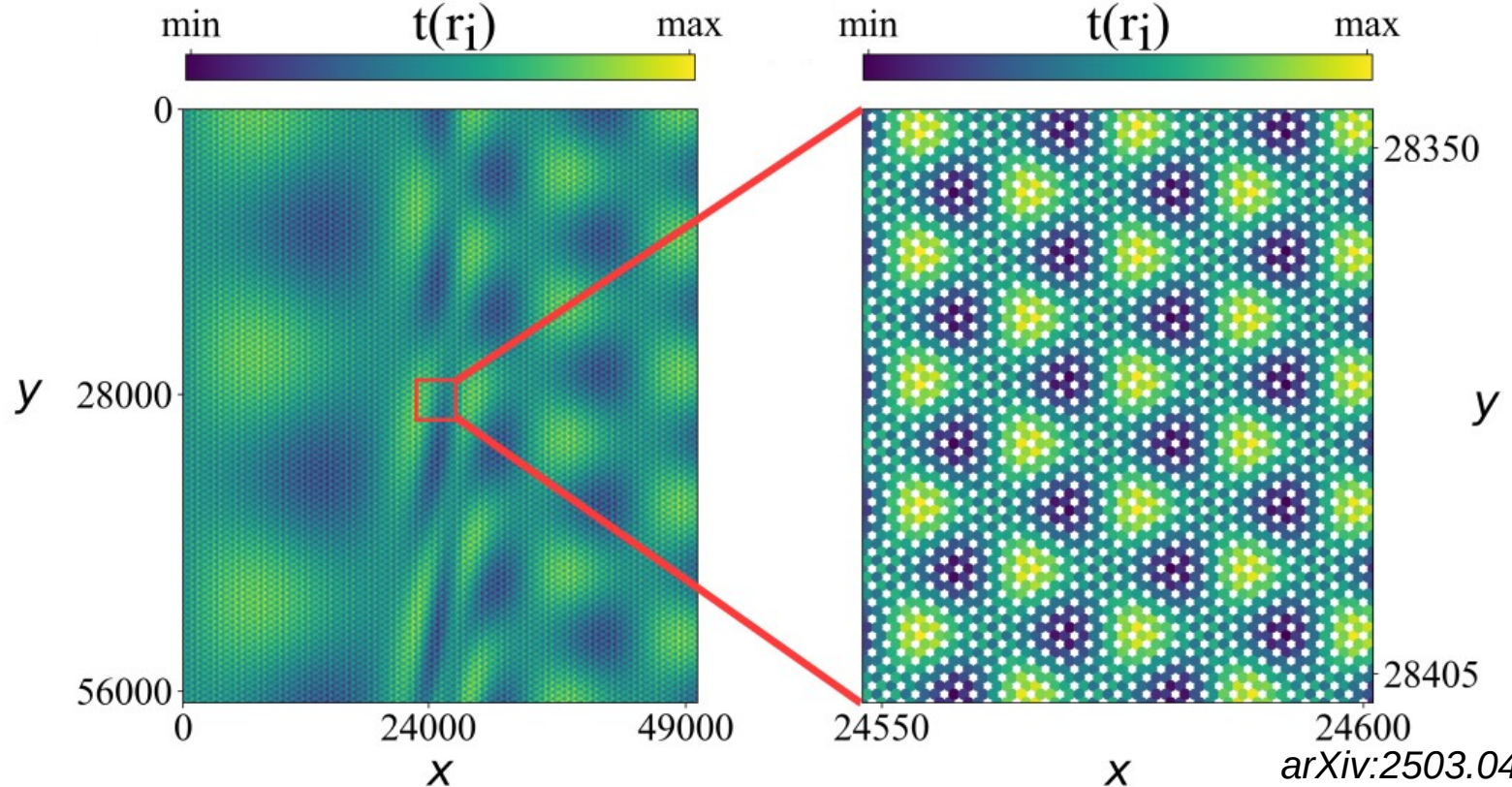
Two-dimensional billion size interacting super-moire

Selfconsistent symmetry broken order (magnetization)



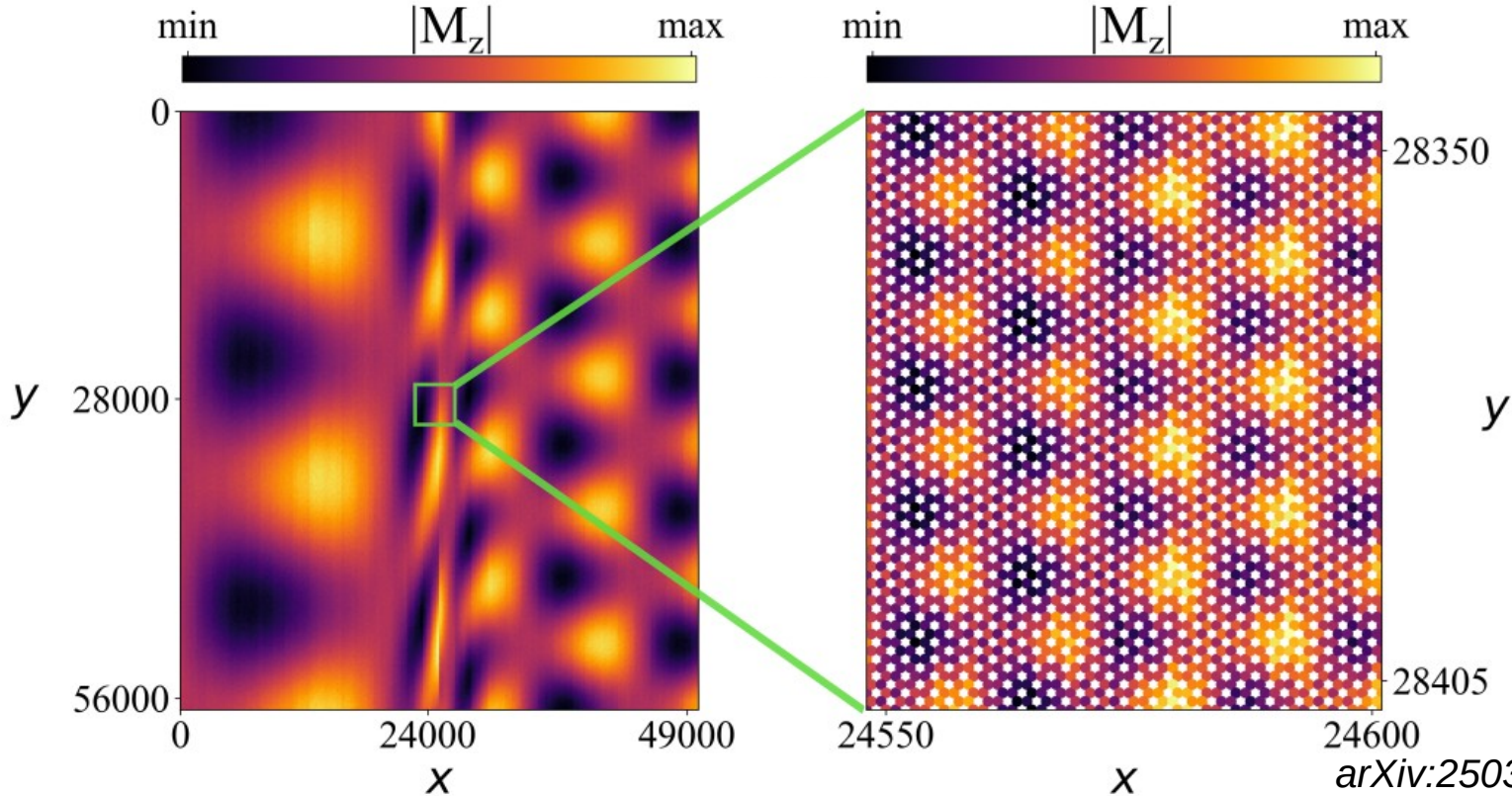
Two-dimensional billion size interacting super-moire

Modulation fo the Hamiltonian



Two-dimensional billion size interacting super-moire

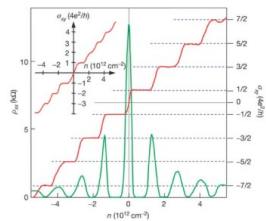
Selfconsistent symmetry broken order (magnetization)



Tensor networks for topological super-moire materials

Topological quantum matter

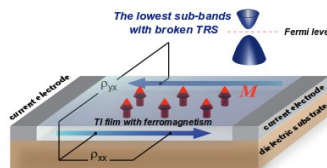
Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

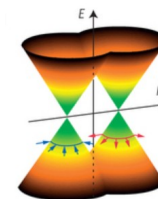
Quantum Anomalous Hall effect



Science 340.6129 (2013): 167-170

Cr-doped
(Bi,Sb)₂Te₃

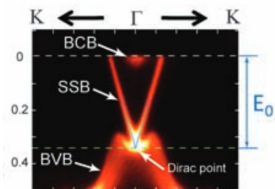
Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

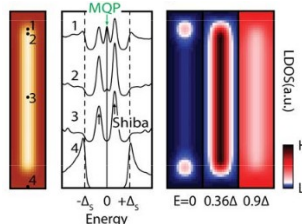
Quantum Spin Hall effect



Bi₂Se₃

Science 325.5937 (2009): 178-181

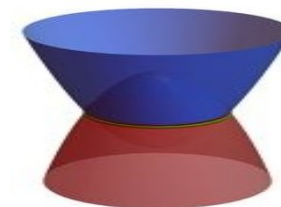
Topological superconductor



Fe@Pb

Science 346.6209 (2014): 602-607

Nodal line semimetals

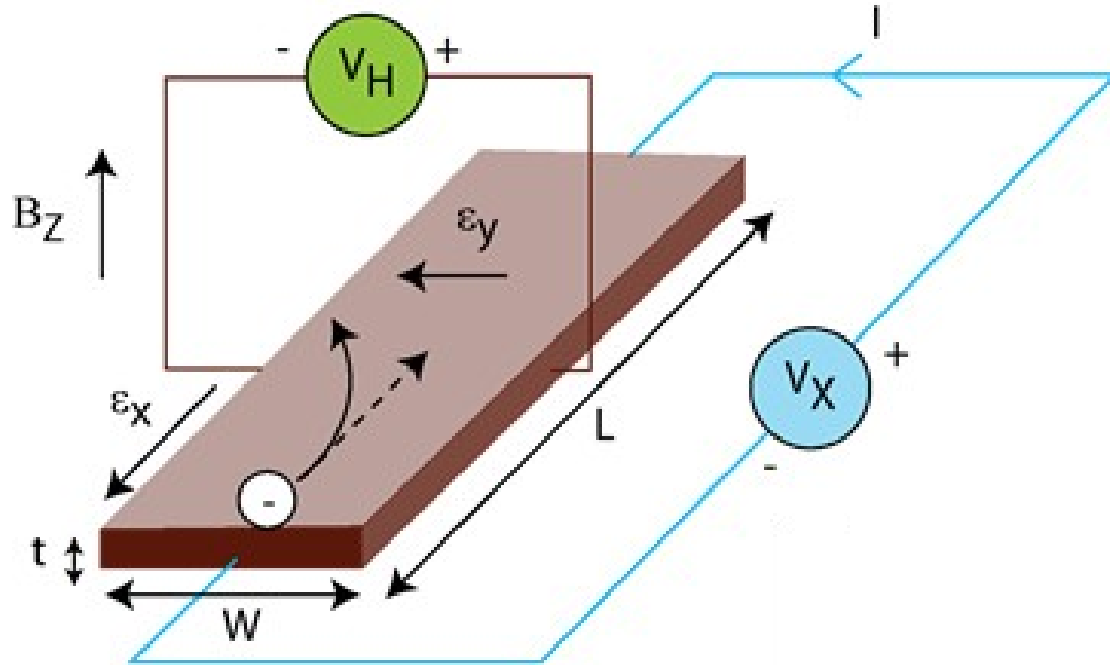


Mg₃Bi₂

Advanced Science 6.4 (2019): 1800897

All these states are described by effective isolated single-particle physics

The Hall effect



$$J_x = \sigma_{xy} V_y$$

Hall conductivity

Measure the current perpendicular to a voltage

The Hall conductivity

The Hall conductivity is obtained as $\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k}$

Berry curvature

$$\Omega_{\alpha} = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}$$

Berry connection

$$A_{\mu}^{\alpha} = i \langle \partial_{k_{\mu}} \Psi_{\alpha} | \Psi_{\alpha} \rangle$$

$$\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k} = \sum_{\alpha} C_{\alpha} = C \longleftarrow \text{Chern number}$$

The transverse conductivity is a topological invariant

This means, it must take integer values regardless of defects in an insulator

Topological invariant in a Hamiltonian

We can classify Hamiltonians according to topological invariants

Hamiltonian

$$H(\mathbf{k})$$

Wavefunction

$$|\Psi(\mathbf{k})\rangle$$

$$C = \int_{BZ} \Omega d^2\mathbf{k}$$

Topological invariant
(Chern number)

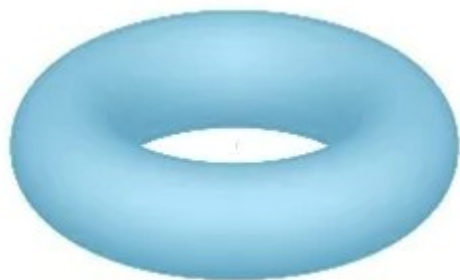
$$\Omega = \nabla \times \mathbf{A}|_z$$

Metric
(Berry curvature)

The role of a topological invariant

**Hamiltonians with different topological invariants
can not be deformed one to another**

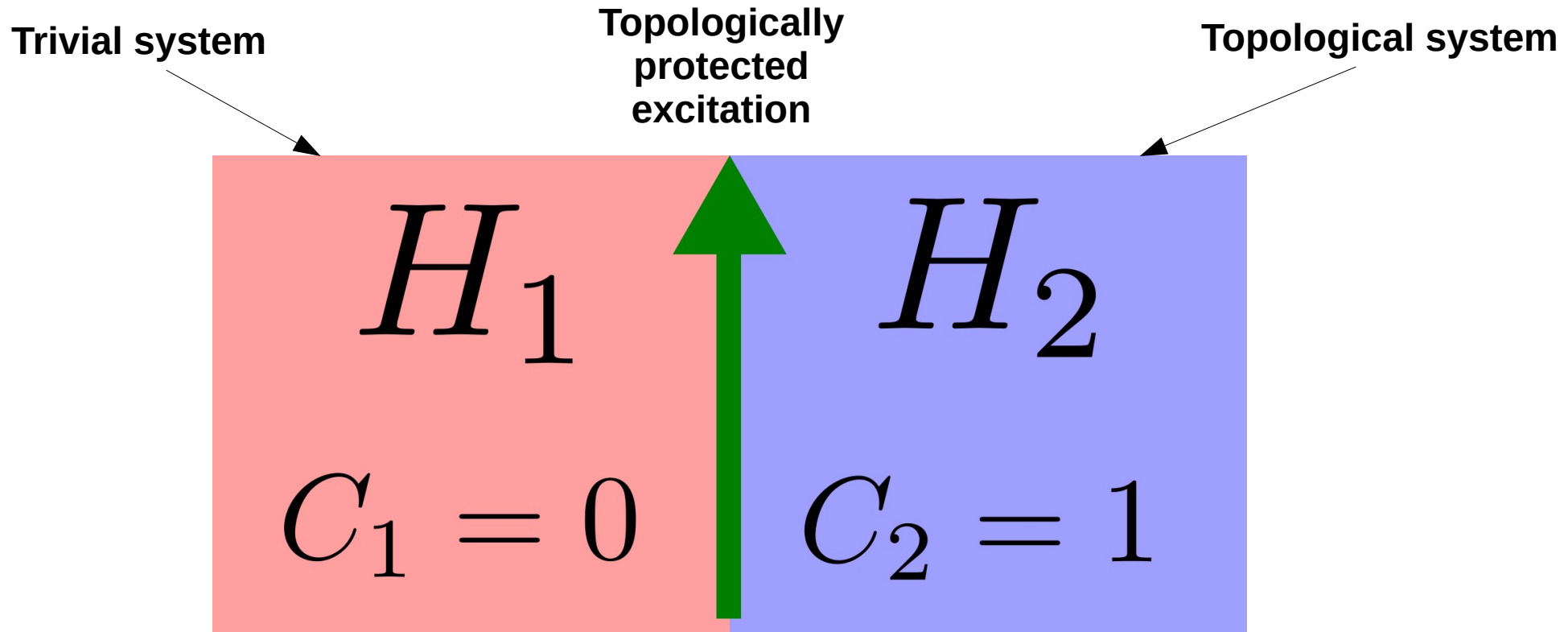
$$C = 1$$



$$C = 2$$



The consequence of different topological invariants



Topological excitations appear between topologically different systems

Real space topological invariants

Topological invariants are often defined in reciprocal space

$$C = \int_{BZ} \Omega d^2 \mathbf{k}$$

$$\Omega_{\alpha} = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}$$

$$A_{\mu}^{\alpha} = i \langle \partial_{k_{\mu}} \Psi_{\alpha} | \Psi_{\alpha} \rangle$$

Is it possible to define them in real space?

Real-space Chern number

$$C_{\alpha} = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

$$\hat{P} = \int_{-\infty}^{\epsilon_F} \delta(\omega - H) d\omega$$

$$\hat{Q} = 1 - \hat{P}$$

Kernel polynomial for topological invariants in real-space

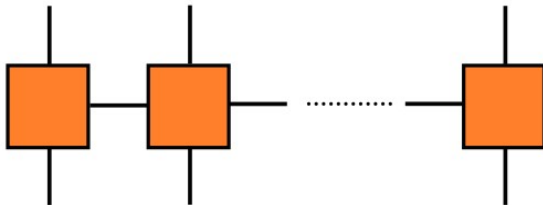
The density matrix of a super-moire system can be expressed as a tensor network

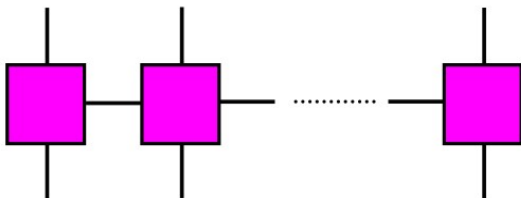
$$\hat{\mathcal{P}} = \int_{-\infty}^{\varepsilon_F} \delta(\omega - \hat{H}) d\omega = \sum \Xi_{s_1, s'_1}^{(1)} \Xi_{s_2, s'_2}^{(2)} \cdots \Xi_{s_L, s'_L}^{(L)} |s\rangle \langle s'|$$

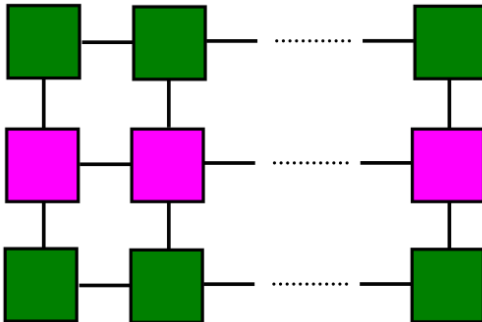
With a kernel polynomial tensor network algorithm

$$\hat{\mathcal{P}} = \sum_n T_n(\hat{\mathcal{H}}) \int_{-\infty}^{\varepsilon_F} d\omega \frac{T_n(\omega)}{\sqrt{1 - \omega^2}},$$
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$
$$T_0 = 1 \text{ and } T_1(x) = x$$

Computing Chern numbers with tensor networks

Density matrix as tensor network $\hat{P} = \int_{-\infty}^{\varepsilon_F} \delta(\omega - \hat{H}) d\omega =$ 

Topological marker as tensor network $\hat{\Gamma} = \hat{Q}\hat{X}\hat{P}\hat{Y}\hat{Q} - \hat{P}\hat{X}\hat{Q}\hat{Y}\hat{P} =$ 

Chern number from tensor network contraction $C_\alpha = 2\pi i \langle \alpha | \hat{\Gamma} | \alpha \rangle =$ 

Tensor network topological marker

Real-space Chern number (Chern marker)

$$C_{\alpha} = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

Tensor-network Chern marker

$$\hat{C}_{\alpha} = 2\pi i \left(\begin{array}{c} \hat{Q} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{P} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{Q} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} \hat{P} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{Q} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{P} \end{array} \right)$$

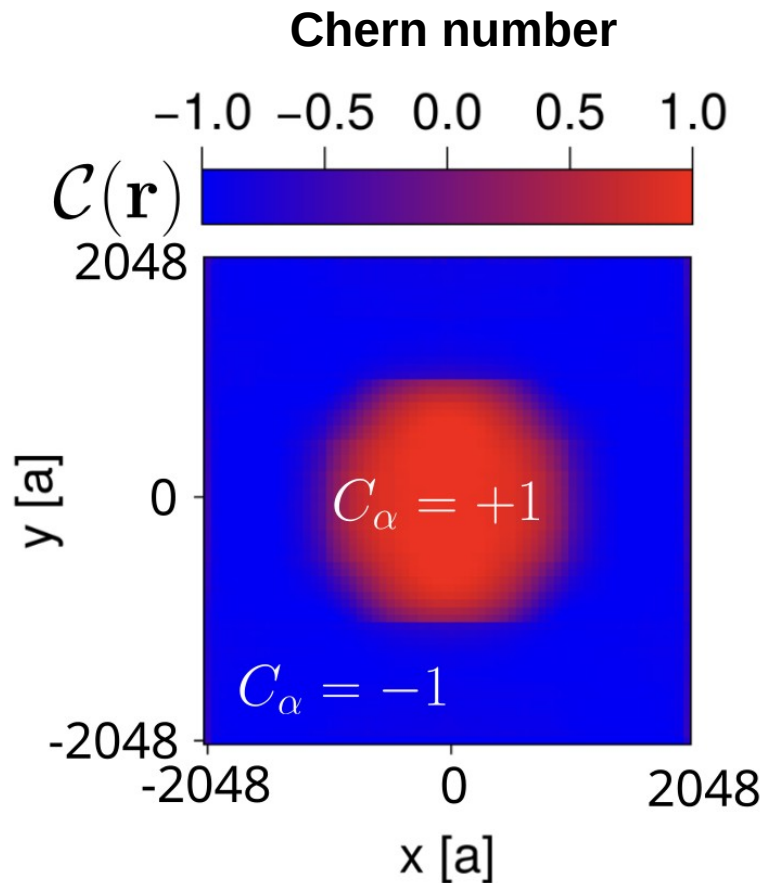
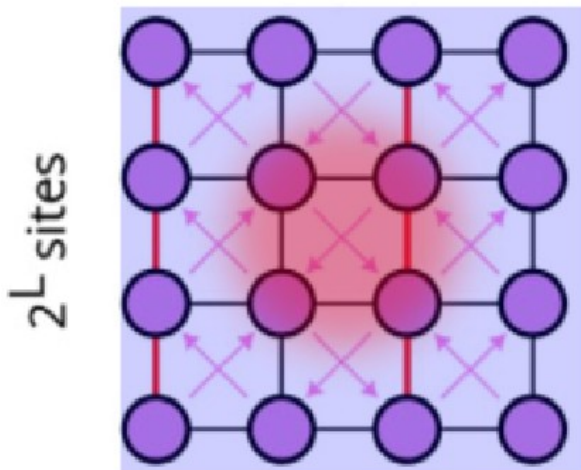
Topological domain with tensor networks

arXiv:2506.05230 (2025)

Topological marker

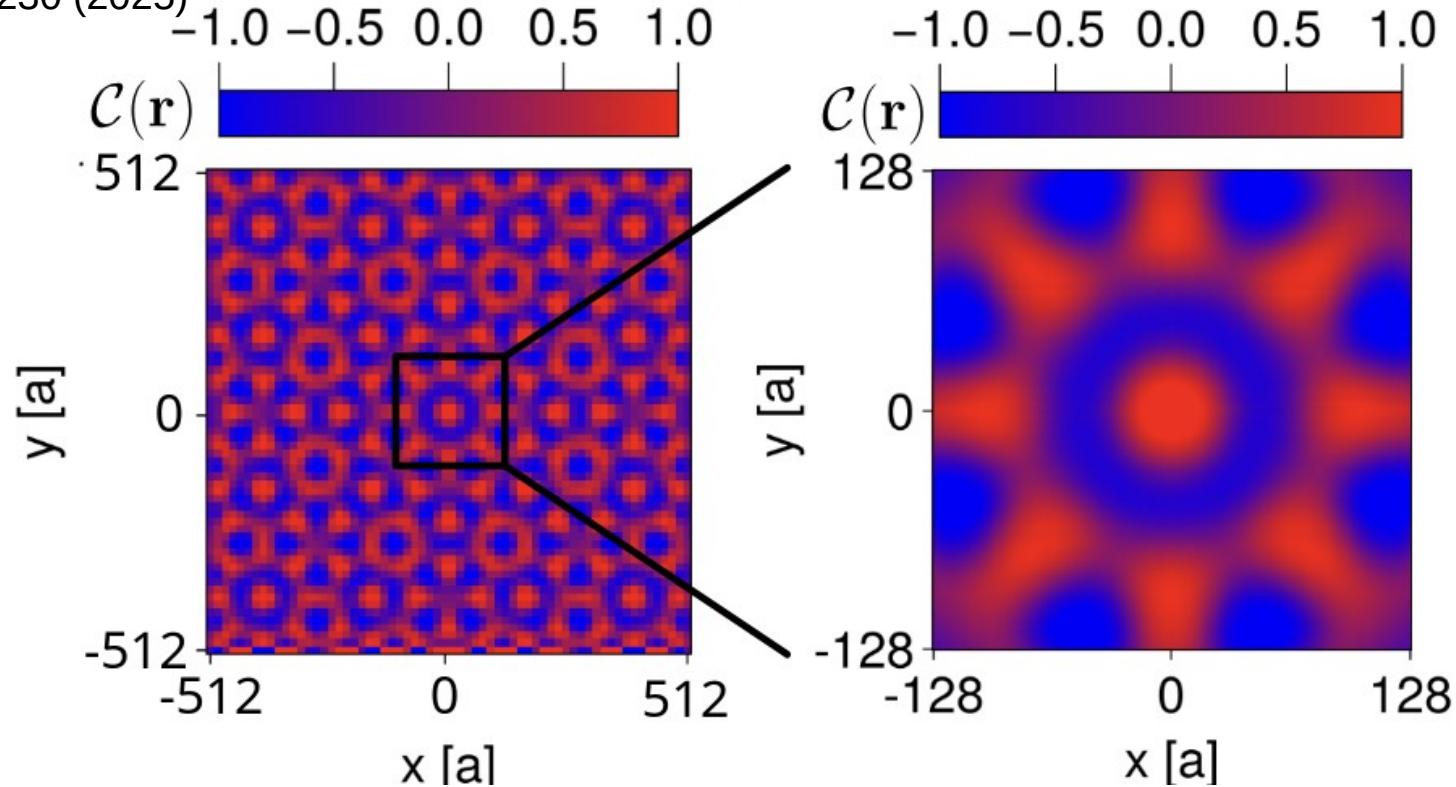
$$C_\alpha = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

**In a spatially modulated
topological Hamiltonian**



Chern mosaic in a 8-fold topological quasicrystal

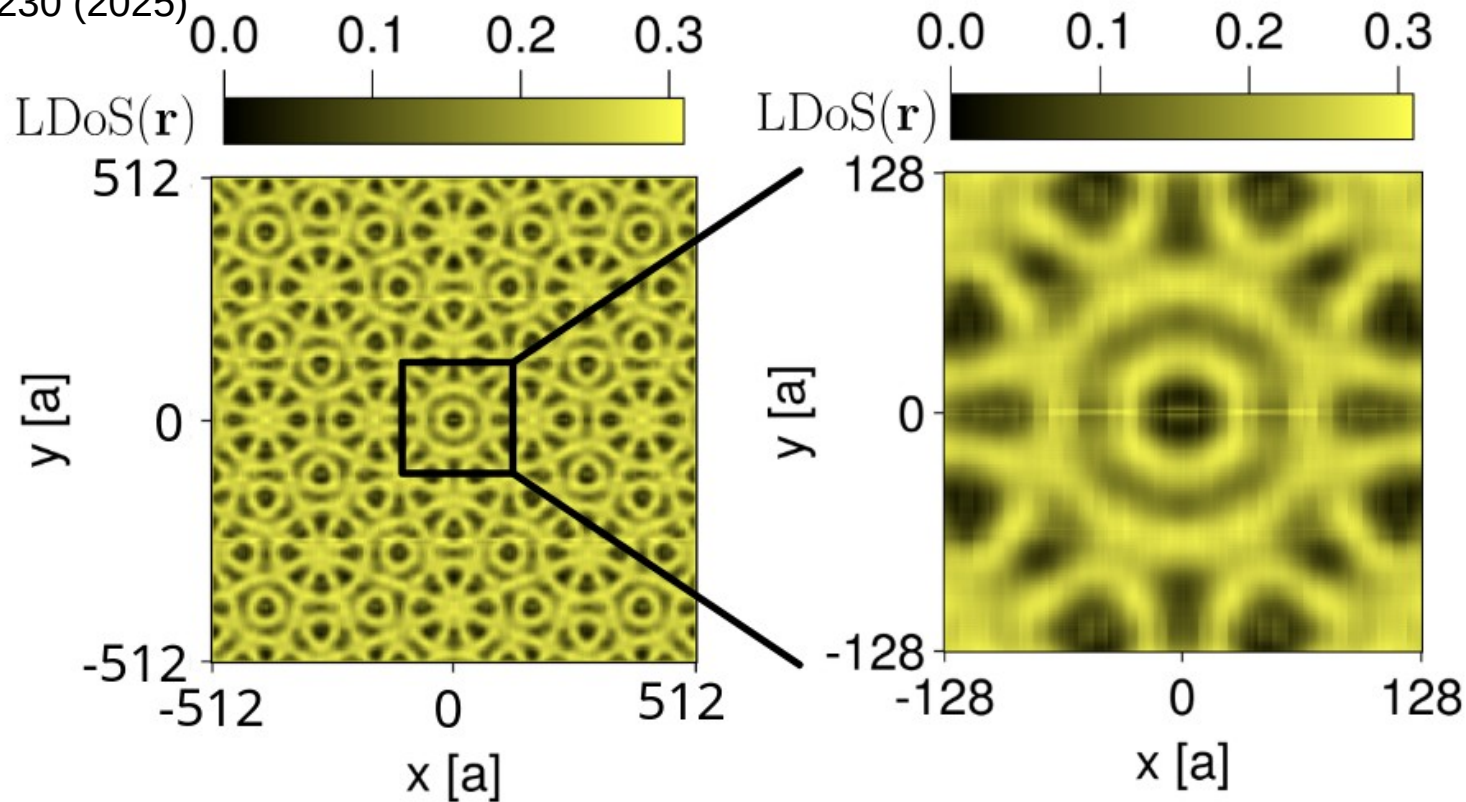
arXiv:2506.05230 (2025)



Domains with different topological invariants can be directly computed in real space

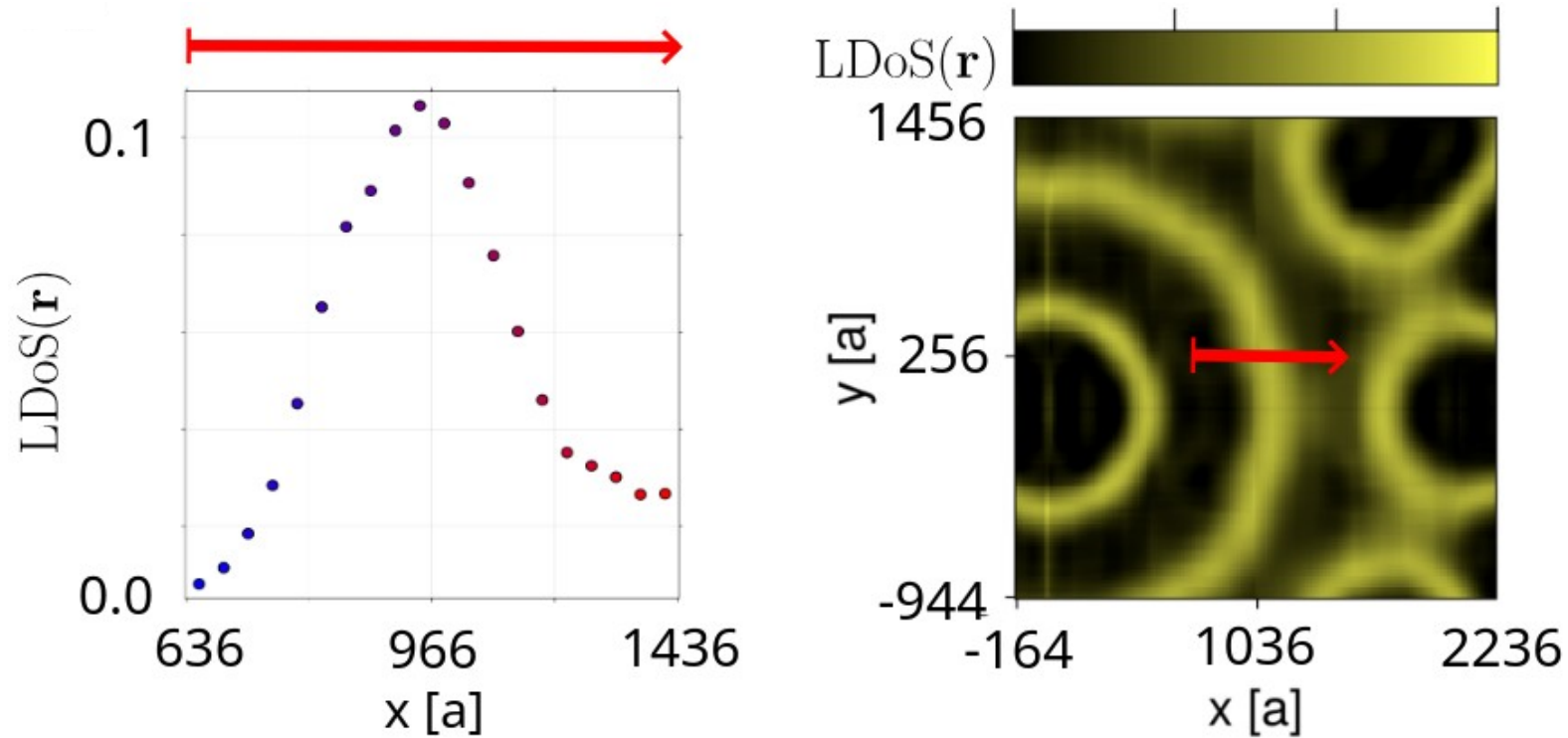
Topological modes in a 8-fold topological quasicrystal

arXiv:2506.05230 (2025)



Topological modes appear between topological domains in real space

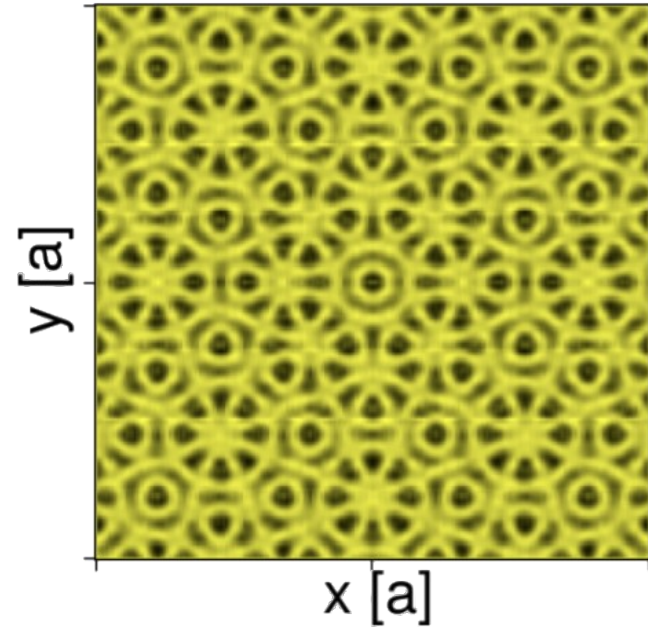
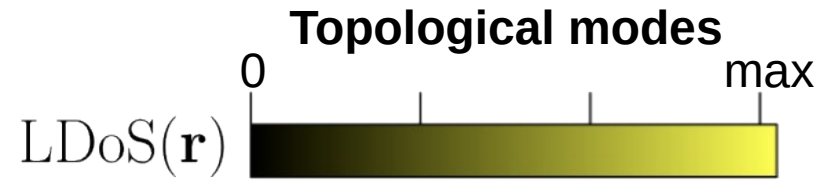
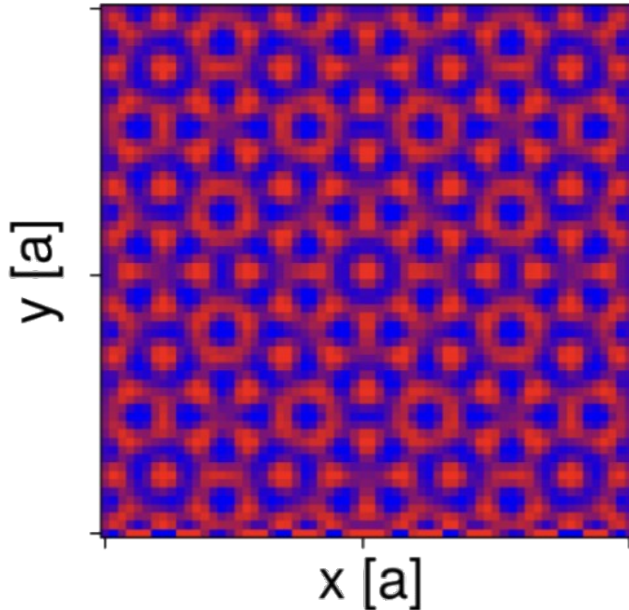
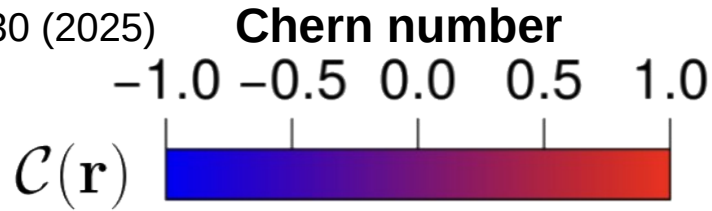
Zero modes across topological quasicrystalline domain



The real space distribution of topological states can be mapped in real space

Chern mosaic and topological modes in a quasicrystal

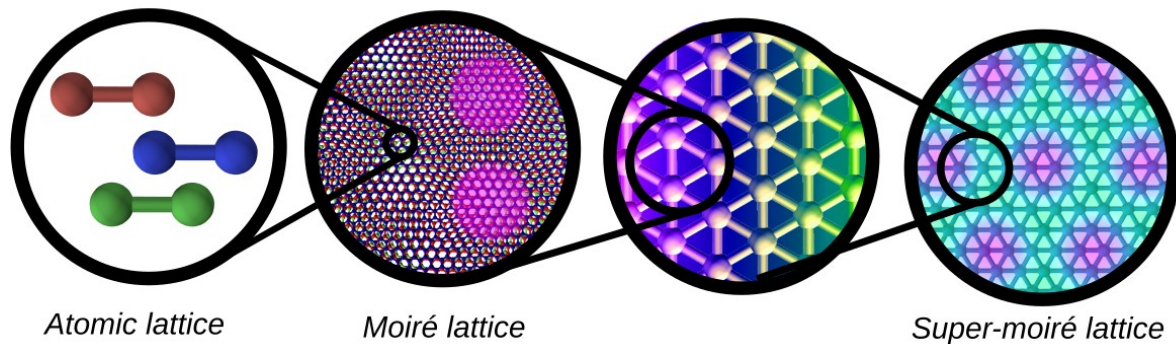
arXiv:2506.05230 (2025)



Tensor networks allow computing topology in exceptionally large super-moire systems

Summary

Tensor networks allows solving large electronic structure problems, reaching the regime required for super-moire materials



2D Materials 12 (1), 015018 (2025)
arXiv:2503.04373 (2025)
arXiv:2506.05230 (2025)

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[orange squares]} = t_{\alpha\beta} + \chi_{\alpha\beta} \left(\text{[cyan squares]} \right)$$

$$\langle c_{\alpha}^{\dagger} c_{\beta} \rangle \equiv \text{[cyan squares]} = \sum_n \lambda_n T_n \left(\text{[orange squares]} \right)$$

$$\hat{C}_{\alpha} = 2\pi i \left(\begin{array}{c} \hat{Q} \\ \hat{Y}_{\alpha}^{\Lambda} \\ \hat{P} \\ \hat{X}_{\alpha}^{\Lambda} \\ \hat{Q} \end{array} \text{[cyan grid]} - \begin{array}{c} \hat{P} \\ \hat{Y}_{\alpha}^{\Lambda} \\ \hat{Q} \\ \hat{X}_{\alpha}^{\Lambda} \\ \hat{P} \end{array} \text{[green grid]} \right)$$