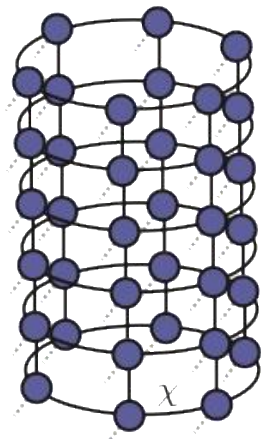
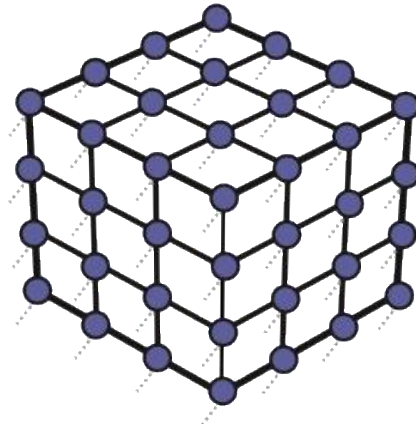


Quantum Simulation with Tensor Networks and Belief Propagation

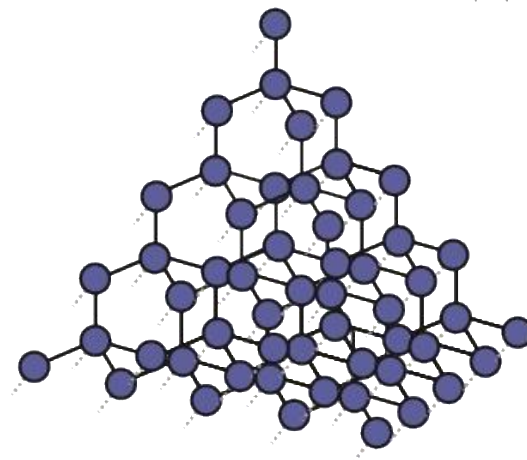
Cylindrical Tensor Network $|\psi\rangle$



Cubic Tensor Network $|\psi\rangle$



Diamond Tensor Network $|\psi\rangle$



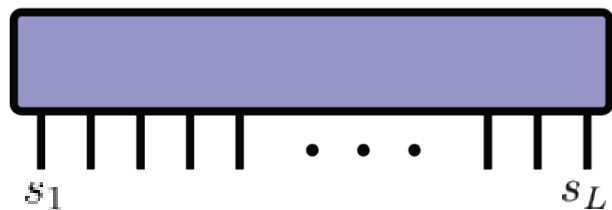
Thursday July 9th 2026

Grenoble

Joseph Tindall
Associate Research Scientist
Centre for Computational Quantum Physics

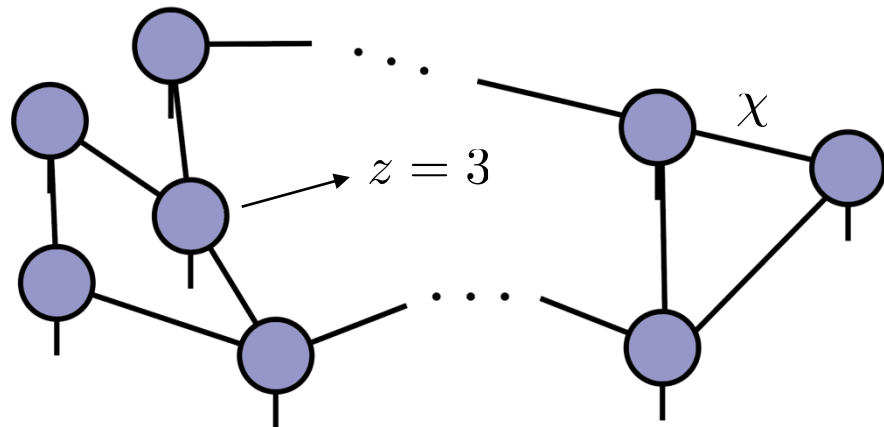
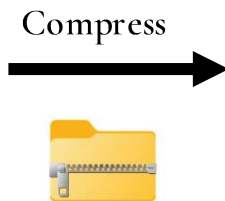
Tensor Networks

A compressed format for high-dimensional tensors



L -dimensional array/ tensor of coefficients

Memory cost: $\mathcal{O}(e^L)$



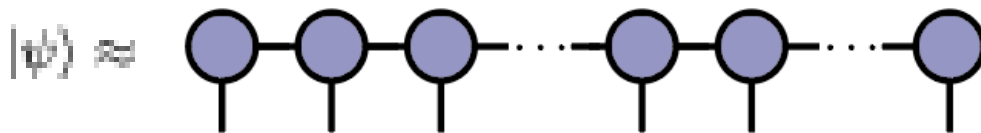
Network of L interconnected tensors

Memory cost: $\mathcal{O}(L\chi^z)$

χ : Maximum dimension of shared indices

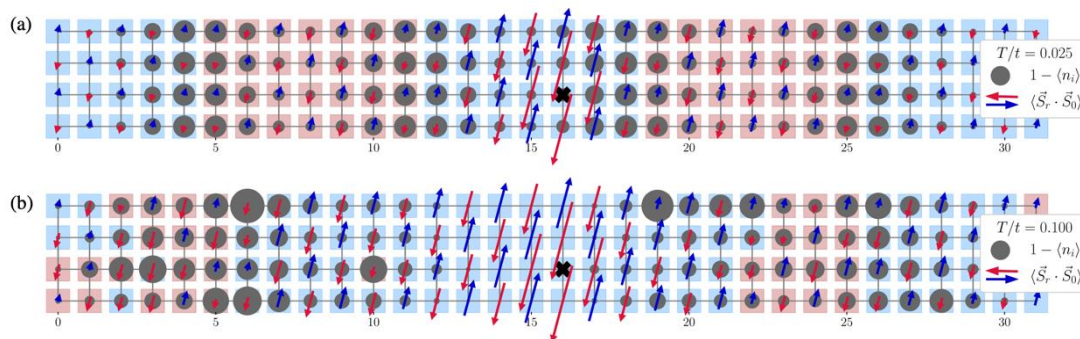
z : Maximum co-ordination number in the network

The Matrix Product State



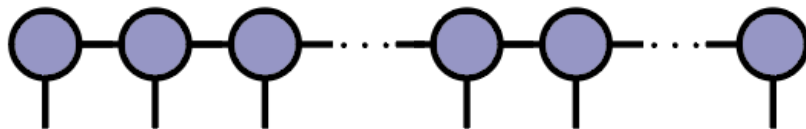
Matrix Product State (MPS)

- Finding ground states of 1D/ Quasi-1D systems
- Short-depth circuit simulation in 1D
- Finite temperature states in 1D

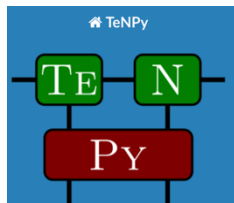
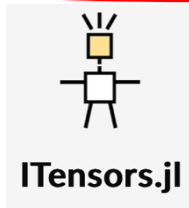


Finite temperature solutions of quasi-2D interacting electrons:
Wietek et al, PRX 11 031007 (2021)

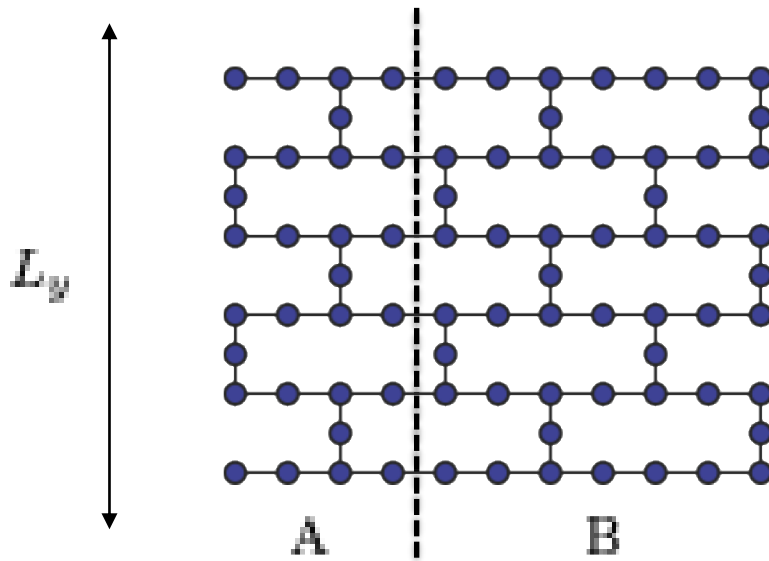
The Matrix Product State



- Can extract properties efficiently from it (norm, entanglement, observables) ✓
- Provably an efficient ansatz for certain problems ✓
- Relatively easy to implement in code ✓
- Long history with stable, well-designed algorithms and codebases ✓
- Can't represent states on large non-1D topologies ✗

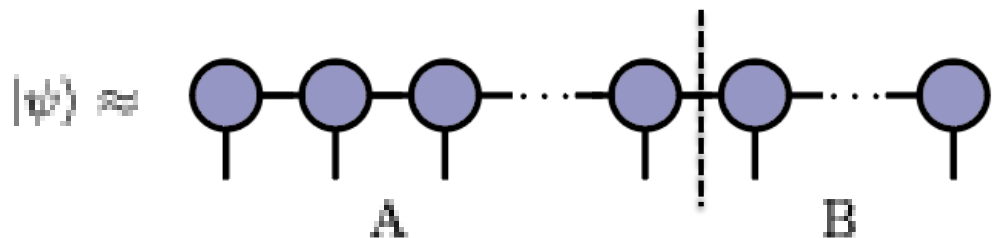


The Problem with Matrix Product States



$$S_{\text{von-Neumann}}(L_y) = \mathcal{O}(L_y)$$

- Area law states (those generated by shallow circuits, short time dynamics, ground states of 2D problems)



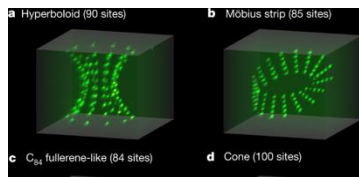
$$\chi \sim \mathcal{O}(\exp(L_y))$$

- MPS bond dimension (classical resources) must go exponentially in system size to capture this

Quantum Devices

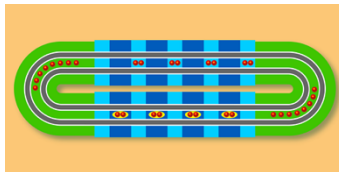
Rydberg Arrays

- Quantum spin quenches



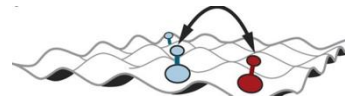
Trapped Ions

- Long-range interactions



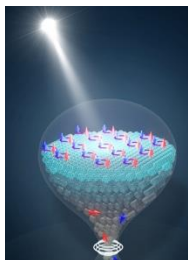
Optical Lattices

- Fermionic and bosonic models



Solid-State Materials

- Ultrafast quantum processes in real materials



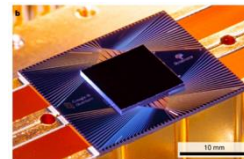
Quantum Annealers

- Dynamical phase transitions



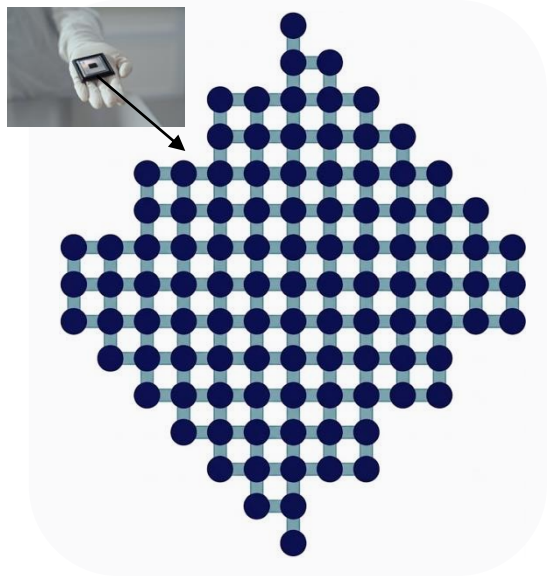
Superconducting Quantum Processors

- Scalable, universal quantum simulators



Higher Dimensional Tensor Networks

We are embracing loopy, higher-dimensional tensor networks in order to understand the quantum wavefunction in higher dimensions and synergize with experimental quantum devices

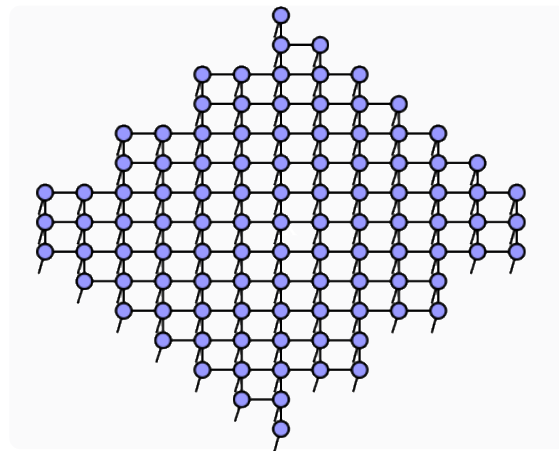


Google's latest Willow processor

Ansatz for the many-body wavefunction $|\psi\rangle$:



Matrix Product State

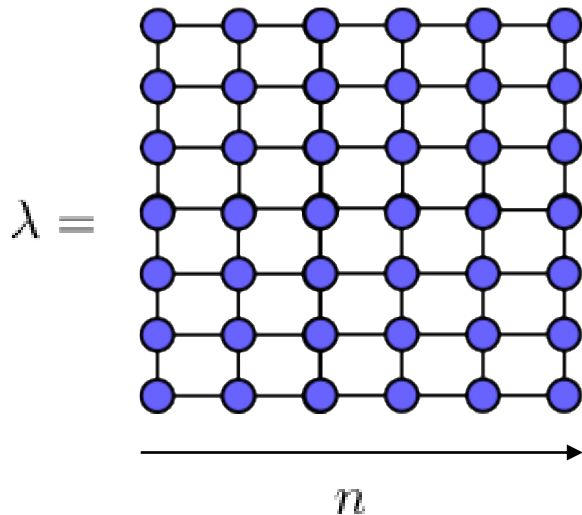


Willow Tensor Network State



Higher Dimensional Tensor Networks

When Tensor Networks have loops we can't efficiently contract them.



Contraction is #P hard and partial contraction is PostBQP.

PRL **96**, 220601 (2006)

PHYSICAL REVIEW LETTERS

week ending
9 JUNE 2006

Criticality, the Area Law, and the Computational Power of Projected Entangled Pair States

F. Verstraete,¹ M. M. Wolf,² D. Perez-Garcia,^{2,3} and J. I. Cirac²

¹*Institute for Quantum Information, Caltech, Pasadena, CA, USA*

²*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Street 1, Garching, D-85748, Germany*

³*Área de Matemática Aplicada, URJC, Móstoles (Madrid), Spain*

(Received 27 January 2006; published 6 June 2006)

PHYSICAL REVIEW RESEARCH **2**, 013010 (2020)

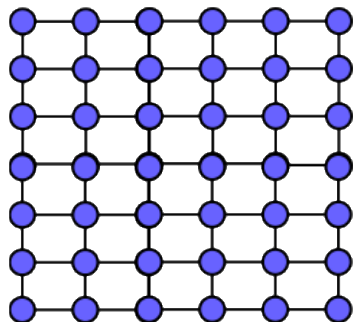
Contracting projected entangled pair states is average-case hard

Jonas Haferkamp, Dominik Hangleiter, Jens Eisert, and Marek Gluza

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany

Higher Dimensional Tensor Networks

There has been significant work on two-dimensional tensor networks



Criticality, the Area Law, and the Computational Power of Projected Entangled Pair States

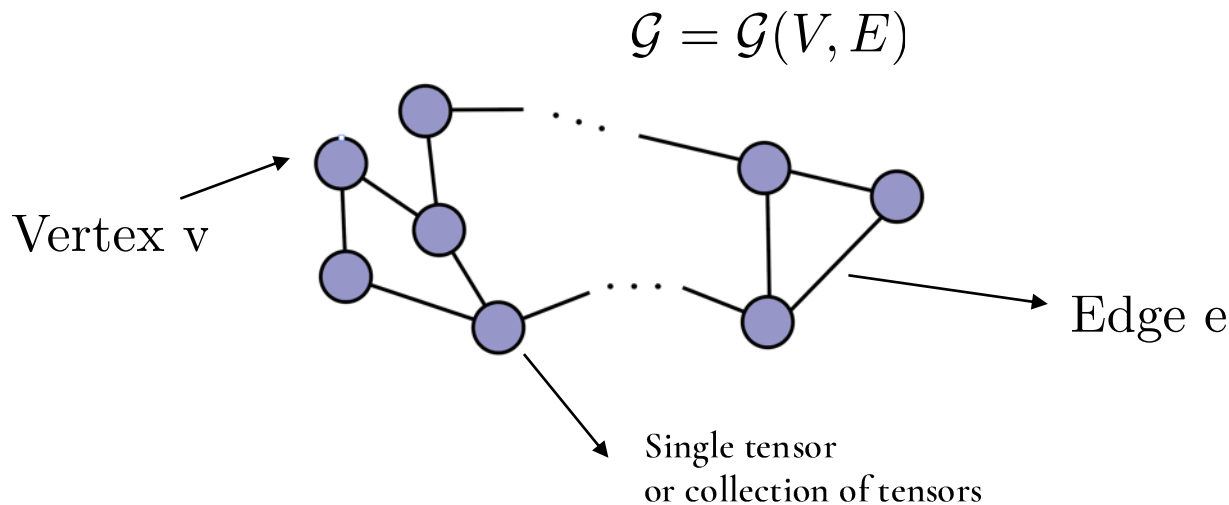
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac
Phys. Rev. Lett. **96**, 220601 – Published 6 June 2006

Algorithms for finite projected entangled pair states

Michael Lubasch, J. Ignacio Cirac, and Mari-Carmen Bañuls
Phys. Rev. B **90**, 064425 – Published 25 August 2014

- Square lattices tensor networks are one of the hardest tensor networks to contract (high tree-width)
- Too many algorithms and software libraries are lattice-based and inflexible
- Many contraction schemes are very computationally expensive due to their focus on precision above all: we must accept approximations

Tensor Networks: A Graphical Perspective



- Accept approximations in contraction to avoid prohibitive polynomial scaling
- Graph-based software and algorithms to avoid being “pigeon-holed” into specific geometries and algorithm choices

Belief Propagation

(1934) Calculating the partition function

Statistical Theory of Superlattices

By H. A. BETHE, H. H. Wills Physical Laboratory, University of Bristol

(Communicated by W. L. Bragg, F.R.S.—Received February 13, 1935)

(1982) Performing statistical inference on graphical models

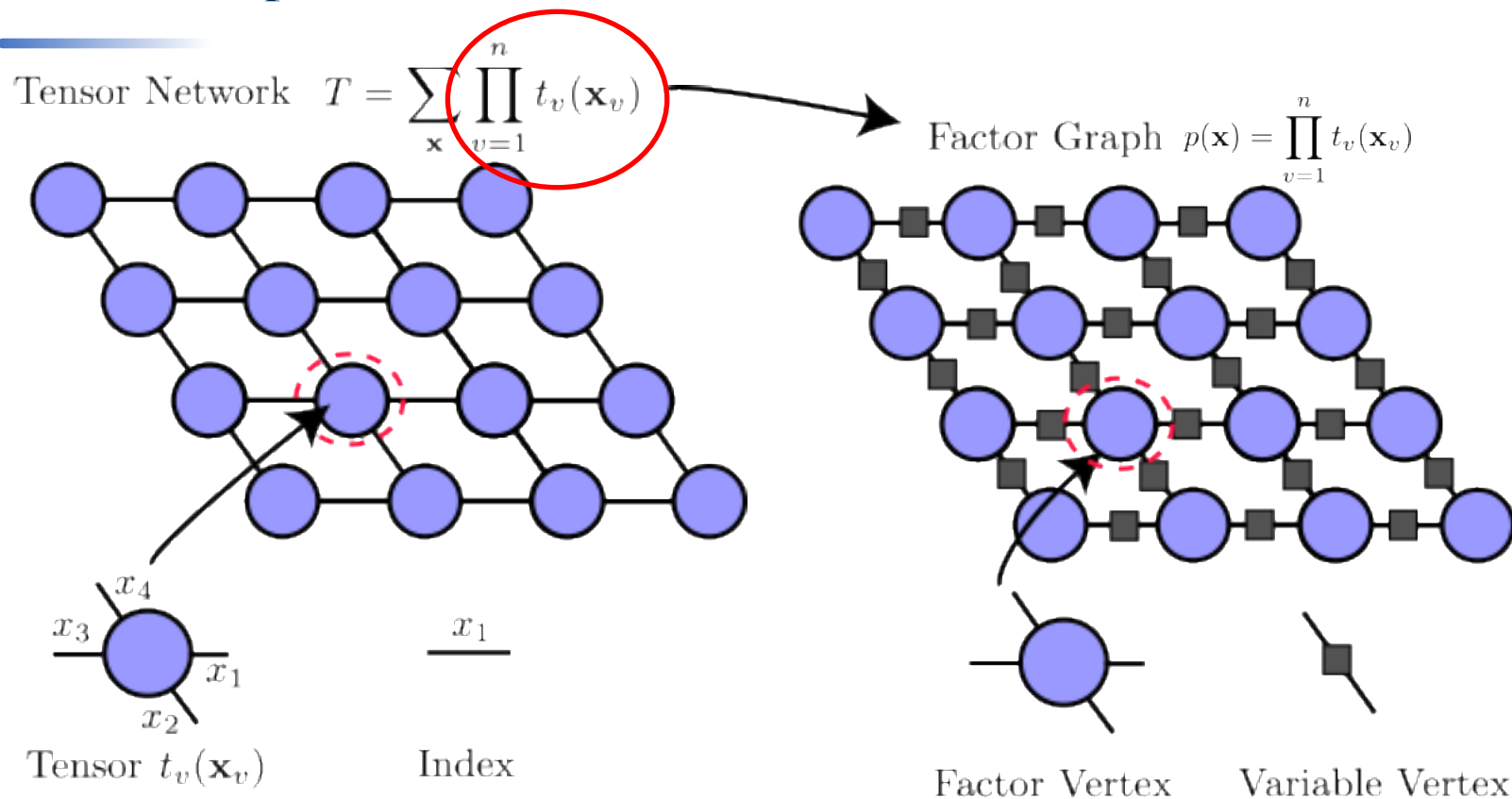
REVEREND BAYES ON INFERENCE ENGINES: A DISTRIBUTED
HIERARCHICAL APPROACH(*)(**)

Judea Pearl
Cognitive Systems Laboratory
School of Engineering and Applied Science
University of California, Los Angeles
90024

(2017-) Contracting tensor networks with Belief Propagation

- Cao and Vontobel, Double-Edge Factor Graphs: Definitions, Properties and Examples, IEEE ITW (2017)
- Alkabetz and Arad, Tensor Network Contraction and the Belief Propagation Algorithm, Phys. Rev. Research 3 023073 (2021)
- Tindall and Fishman, Gauging Tensor Networks with Belief Propagation, SciPost Phys. 15 (6) 222 (2023)

Factor Graphs and Tensor Networks

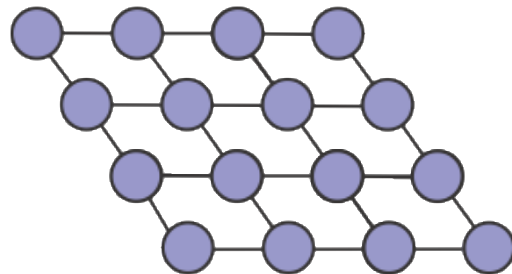


Factor Graphs and Tensor Networks

REVEREND BAYES ON INFERENCE ENGINES: A DISTRIBUTED
HIERARCHICAL APPROACH(*)(**)

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Cognitive Systems Laboratory
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University of California, Los Angeles
90024

Tensor Network $T = \sum_{\mathbf{x}} \prod_{v=1}^n t_v(\mathbf{x}_v)$

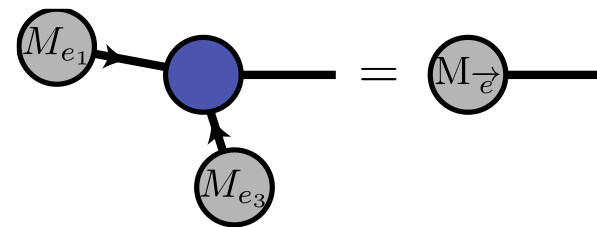
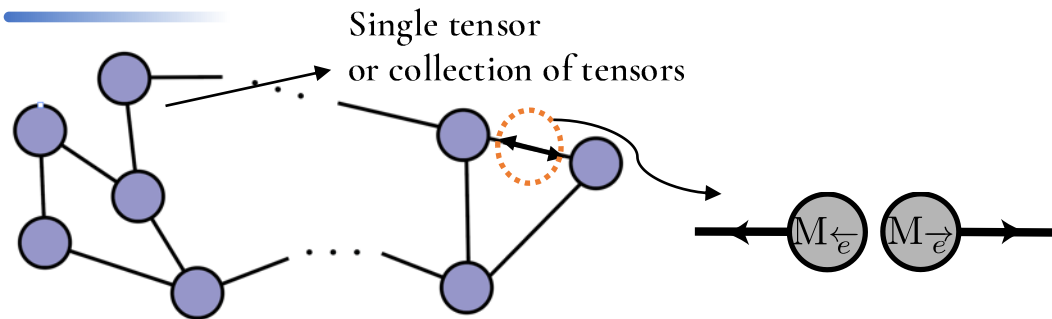


$$p(\mathbf{x}) = \prod_{v=1}^n t_v(\mathbf{x}_v)$$

Marginals of $p(\mathbf{x})$ are directly related to the partial contraction / derivatives of the tensor network (TN) so we can immediately leverage the Judea Pearl BP Algorithm to approximate partial contractions of a TN

$$\frac{\sum_{\mathbf{x} \setminus \mathbf{x}_v} p(\mathbf{x})}{t_v(\mathbf{x}_v)} =$$

Belief Propagation for Tensor Network Contraction

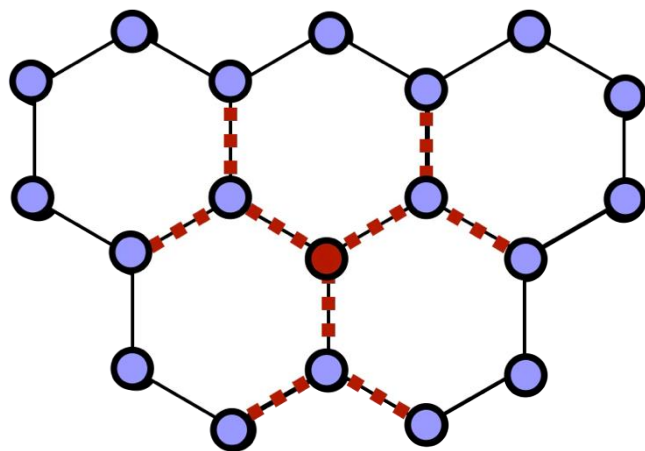


$$\lambda \approx \frac{\prod_{\text{vertices}} \text{[vertex diagram]}}{\prod_{\text{edges}} \text{[edge diagram]}} = e^{F_{\text{Bethe}}}$$

The equation shows the approximation of the scalar λ as the ratio of the product of vertex contributions to the product of edge contributions, which is equal to the exponential of the Bethe free energy F_{Bethe} . The vertex diagram shows a central blue tensor connected to five gray message tensors. The edge diagram shows two gray message tensors connected by a double-headed arrow.

Belief Propagation for Tensor Network Contraction

Approximating the full contraction of the tensor network via the Bethe free energy



$$= \lambda$$

$$\lambda \approx \frac{\prod_{\text{vertices}} \text{diagram}}{\prod_{\text{edges}} \text{diagram}}$$

The numerator diagram shows a central blue node with four incoming arrows from grey nodes. The denominator diagram shows two grey nodes connected by a double-headed arrow.

$$\xi = 2 \quad l_{\min} = 6$$

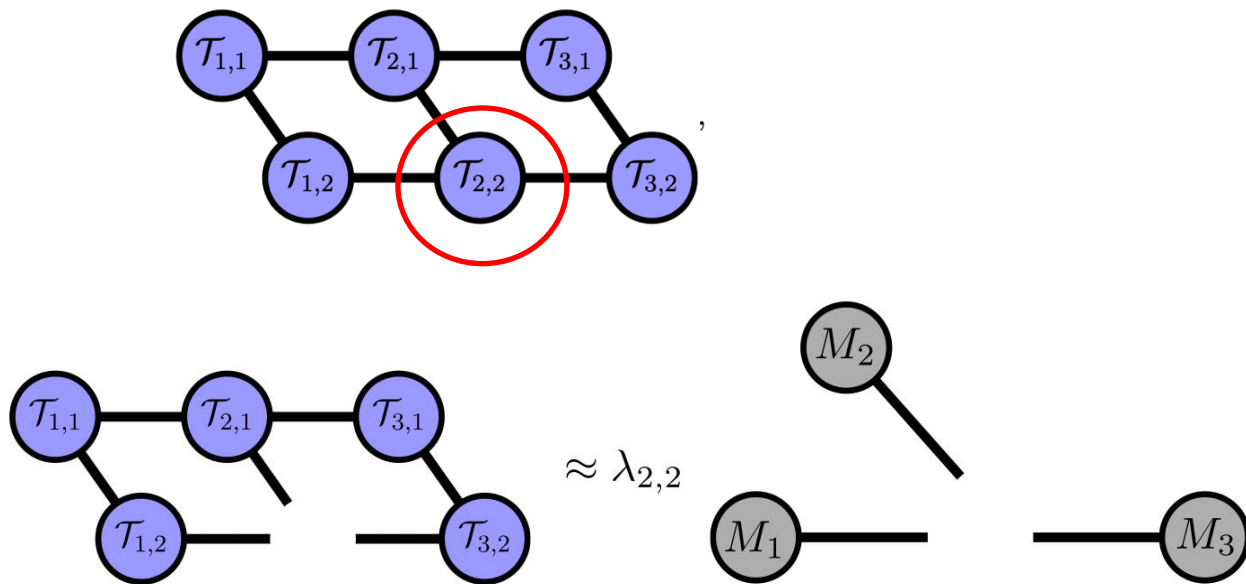
- Efficiently computable
- Applicable to a tensor network of any structure
- Exact on tree tensor networks

BP Error

$$\epsilon_{\text{BP}} \sim e^{-a \frac{l_{\min}}{\xi}}$$

Belief Propagation for Tensor Network Contraction

Can be understood as a mean-field approximation on the environment of subregions of the tensor network



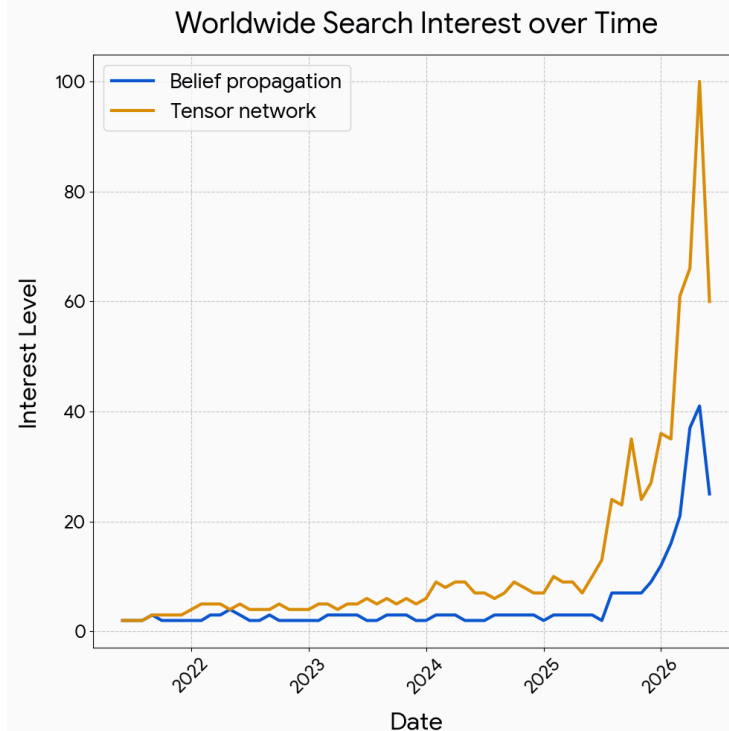
BP + Tensor Networks = “Quantum Belief Propagation”

Applications

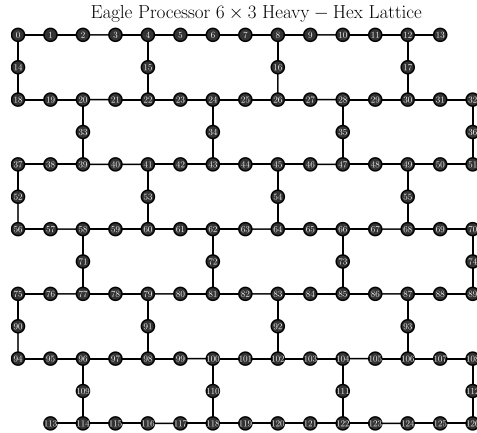
- Quantum circuit simulation and benchmarking
- Equilibrium states of matter (finite and zero temperature)
- Continuous dynamics of quantum systems

Rapidly growing body of research

- A. Alkabetz and I. Arad Phys. Rev. Research 3, 023073 (2021)
- S. Sahu and B. Swingle, arXiv:2206.04701 (2022)
- J. Tindall and M. Fishman, SciPost Physics 15 222 (2023)
- J. Tindall et al, PRX Quantum 5 01038 (2024)
- G. Evenbly, et al, Phys. Rev. Research 8 1 (2026)
- J. Tindall, et al Science 392 6800 (2026)
- S. Midha and Y. F. Zhang, arXiv: 2510.02290 (2025)
- J. Gray et al, arXiv: 2510.05647 (2025),
- G. Evenbly et al, arXiv:2512.10910 (2025)
- S. Midha et al, arXiv:2604.03228 (2026)
- J. Tindall et al, arXiv:2604.24760 (2026)

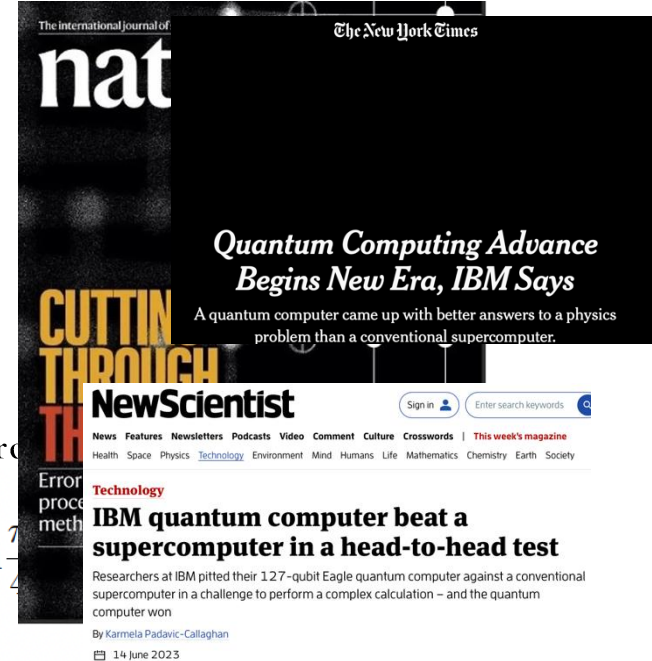
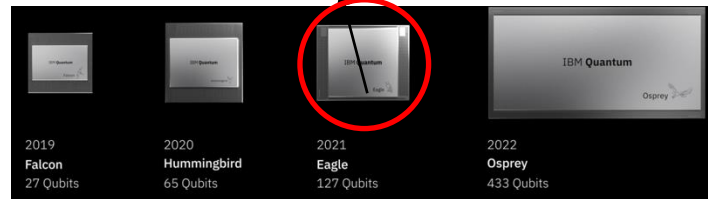
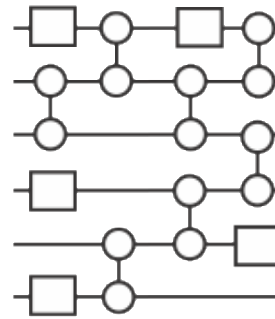


Quantum Circuit Simulation



Quantum Circuit

+



$$U(\theta_h) = \left(\prod_{\langle v, v' \rangle} \exp(i\theta_h \sigma_v \sigma_{v'}) \right)$$

[1] Y. Kim et al, evidence for the quantum utility of quantum computing before fault tolerance, Nature 618 500-505 (2023)

Quantum Circuit Simulation

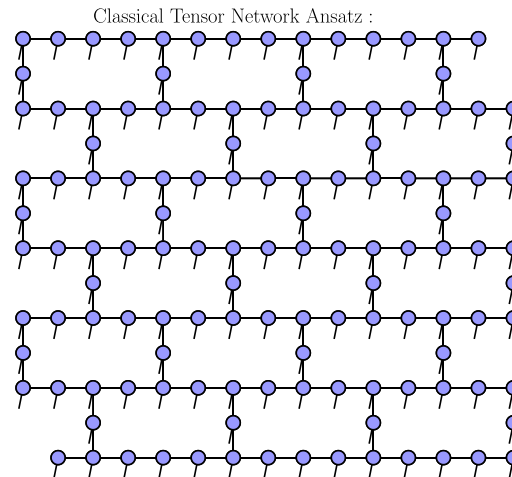
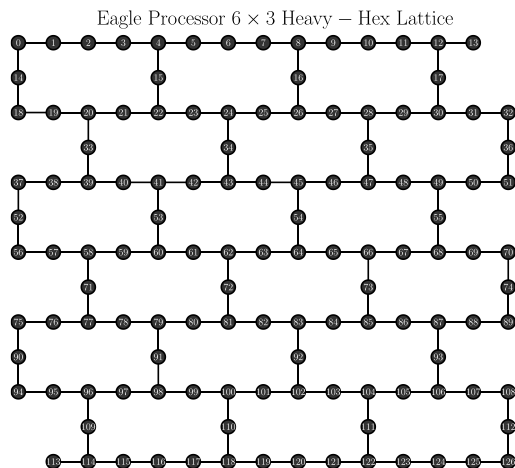
Featured in Physics

Open Access

Efficient Tensor Network Simulation of IBM's Eagle Kicked Ising Experiment

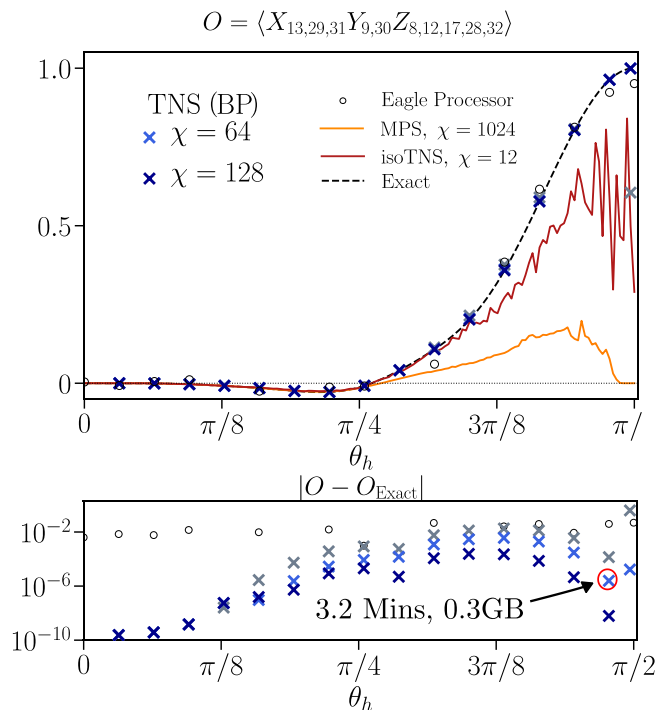
Joseph Tindall, Matthew Fishman, E. Miles Stoudenmire, and Dries Sels
PRX Quantum **5**, 010308 – Published 23 January 2024

PhysICS See Research News: [A Moving Target for Quantum Advantage](#)

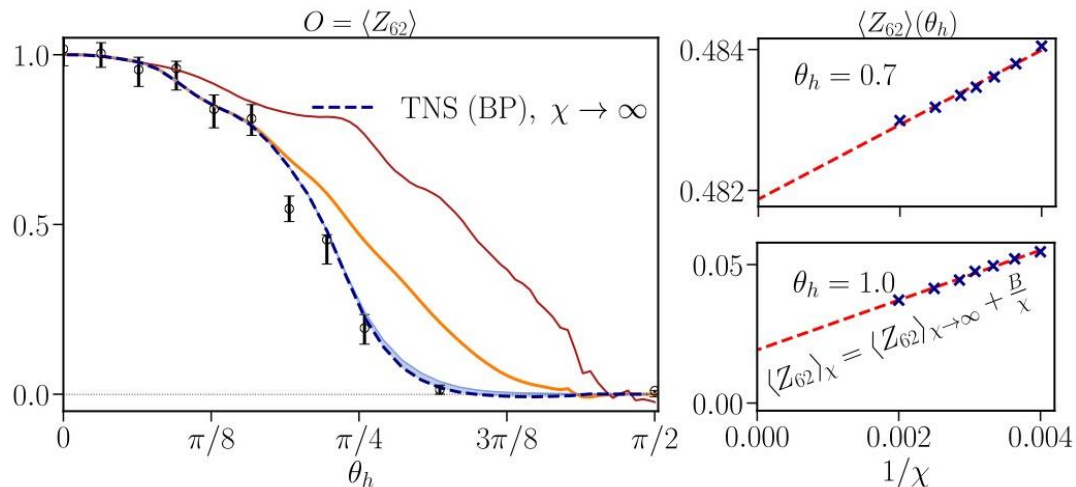


Quantum Circuit Simulation

Quantum BP gets state-of-the-art results over both the quantum computer and other classical methods



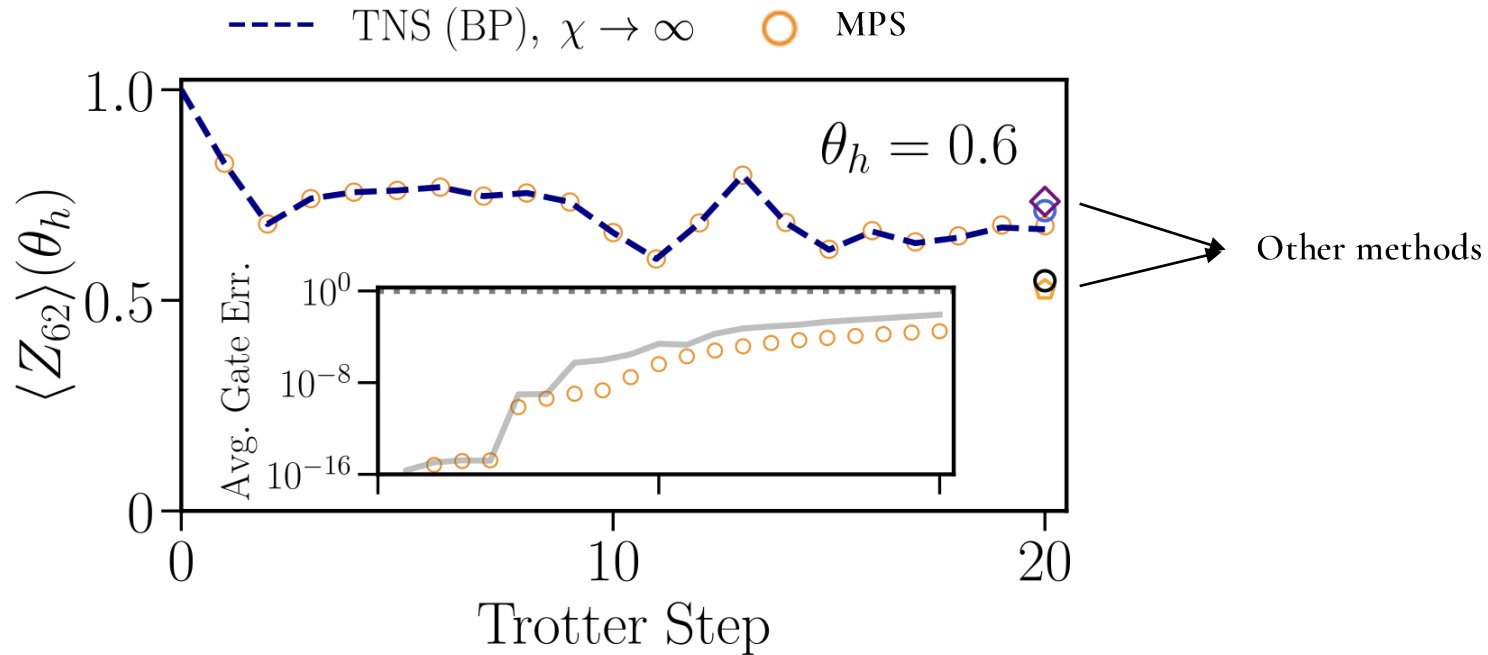
Verifiable Regime



“Unverifiable” Regime

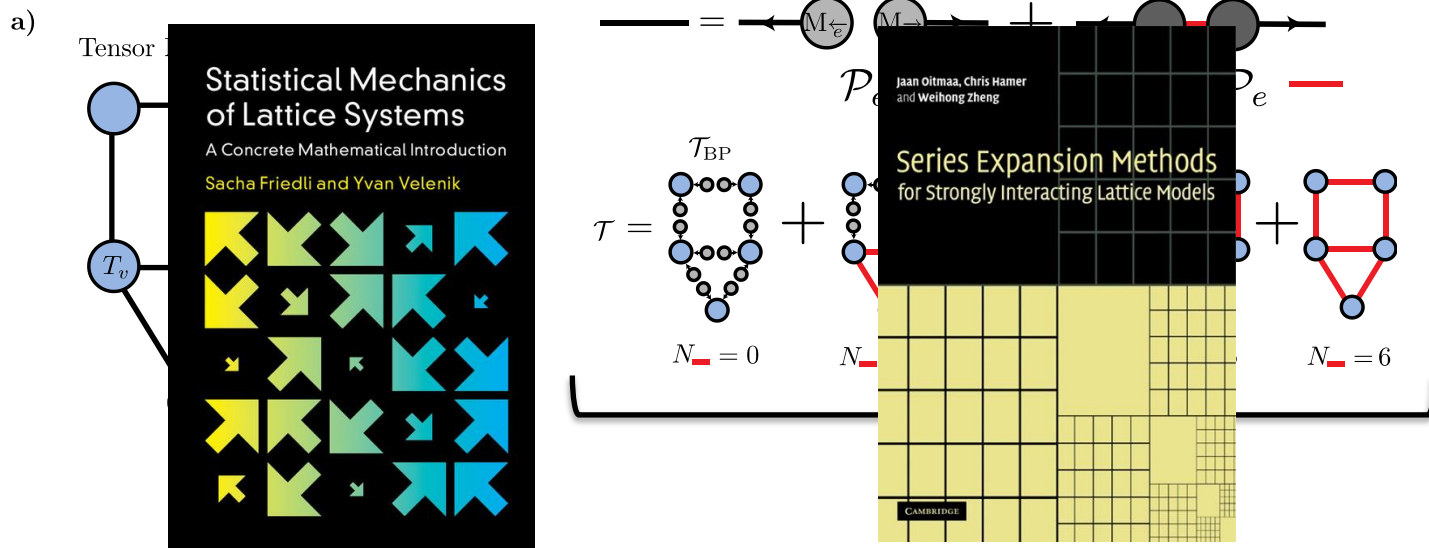
Quantum Circuit Simulation

These results agreed with the ground truth where it can be found



Correcting Belief Propagation – Loop and Cluster Expansions

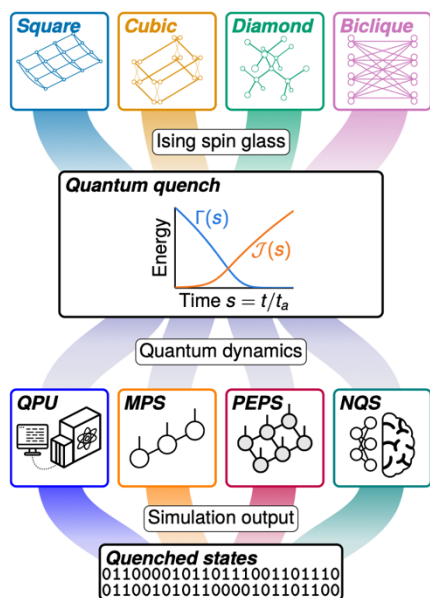
Draw inspiration from cluster and loop expansions in Statistical Mechanics to correct around the BP fixed point



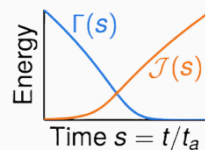
Disordered quantum dynamics in two and three dimensions

Quantum advantage experiment: A. D. King et al, Beyond-classical computation in quantum simulation, Science 10.1126 (2025)

$$H(s) = \Gamma(s) \sum_{\langle i,j \rangle} J_{i,j} \sigma_i^z \sigma_j^z + \mathcal{J}(s) \sum_i \sigma_i^x,$$



Quantum quench

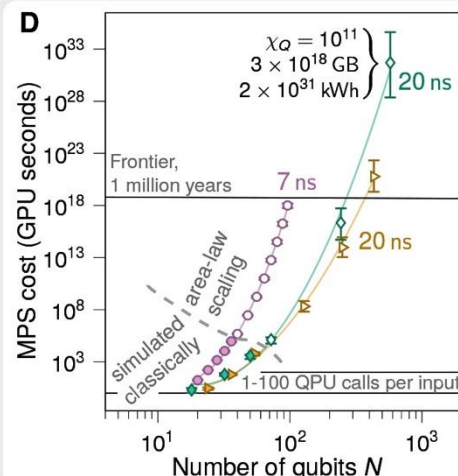


$$\epsilon_c = \sqrt{\frac{\sum_{i>j} (c_{i,j} - \tilde{c}_{i,j})^2}{\sum_{i>j} \tilde{c}_{i,j}^2}}$$

$$c_{i,j} = \langle \sigma_i^z \sigma_j^z \rangle$$

$$\tilde{c}_{i,j} = \langle \sigma_i^z \sigma_j^z \rangle_{\text{Exact}}$$

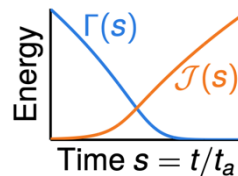
Projected MPS resources to match QPU



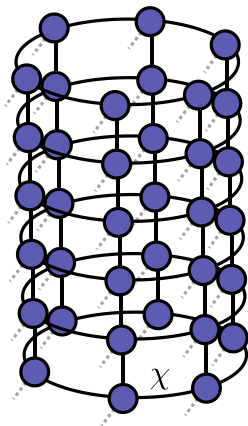
Disordered quantum dynamics in two and three dimensions

Classical simulation: J. Tindall, A. Mello, M. Fishman, M. Stoudenmire and D. Sels, Science 392 6800 (2026)

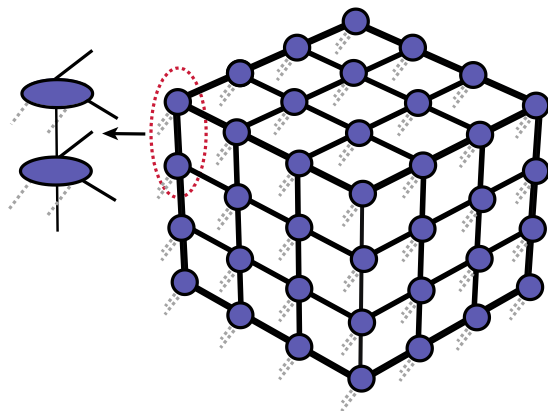
$$H(s) = \Gamma(s) \sum_{\langle i,j \rangle} J_{i,j} \sigma_i^z \sigma_j^z + \mathcal{J}(s) \sum_i \sigma_i^x,$$



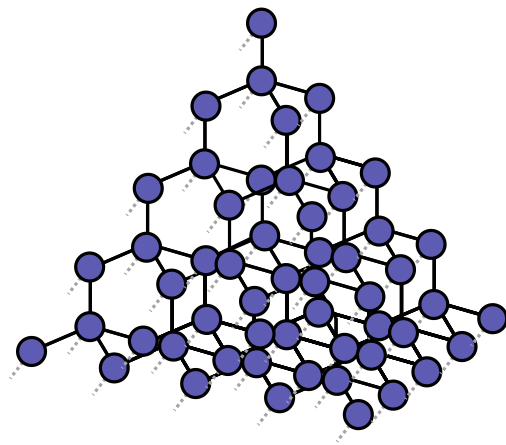
Cylindrical Tensor Network $|\psi\rangle$



Cubic Dimer Tensor Network $|\psi\rangle$

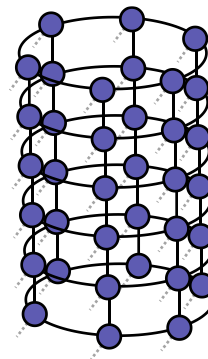
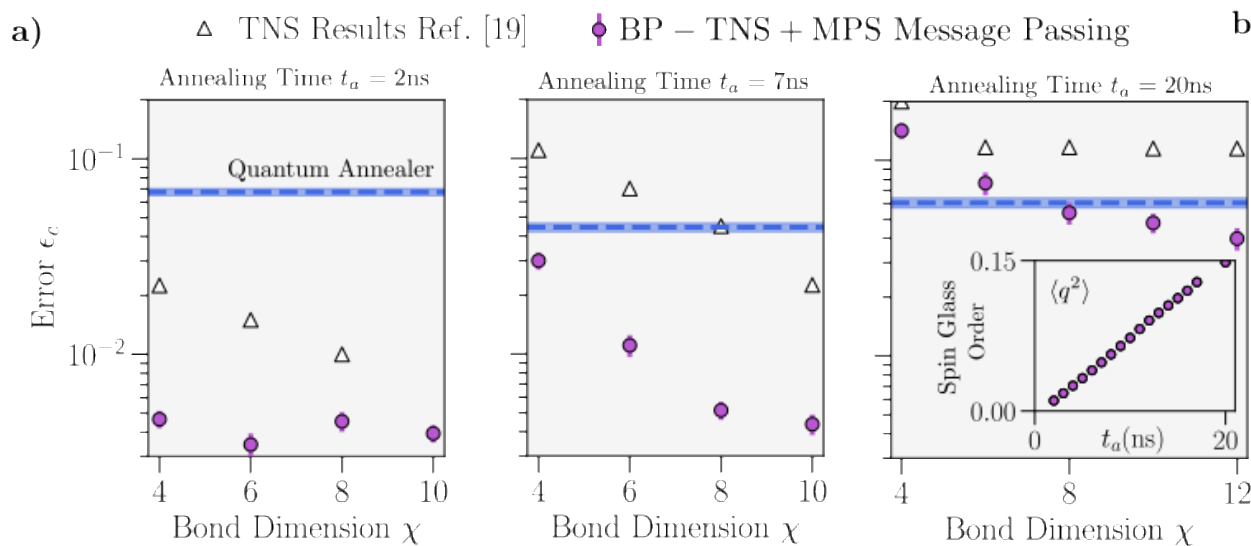


Diamond Cubic Tensor Network $|\psi\rangle$



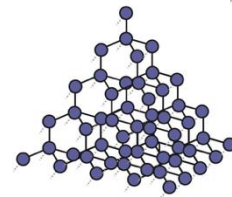
Disordered quantum dynamics in two and three dimensions

Significant improvement compared to evolution + measurement protocols adopted in the original paper

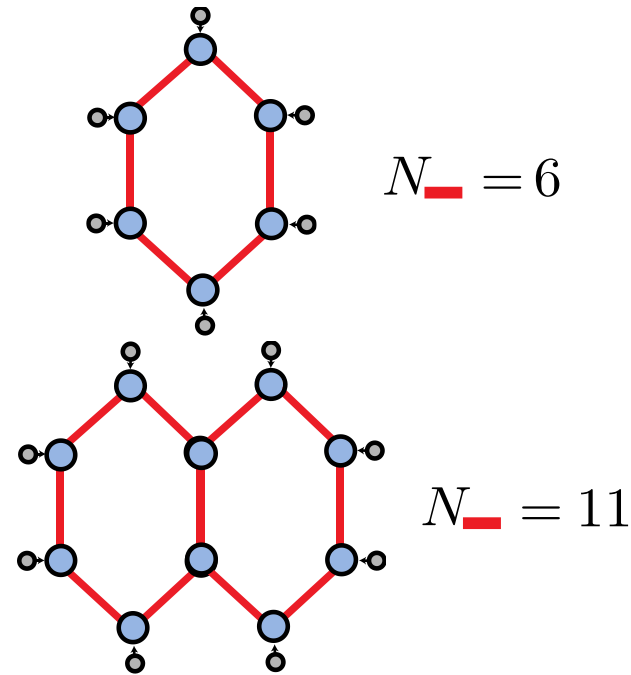
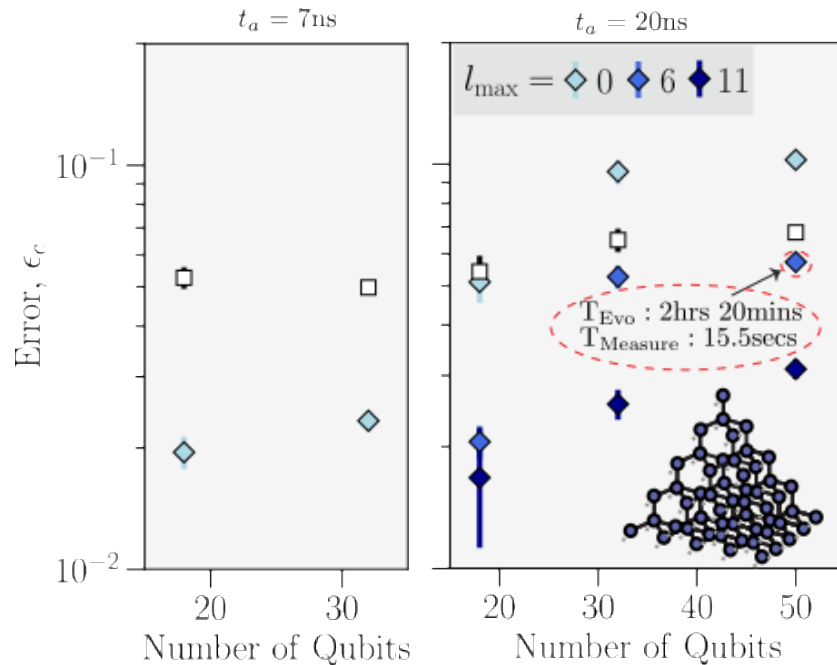


[19] A. D. King et al, Beyond-classical computation in quantum simulation, Science 10.1126 (2025)

Disordered quantum dynamics in two and three dimensions

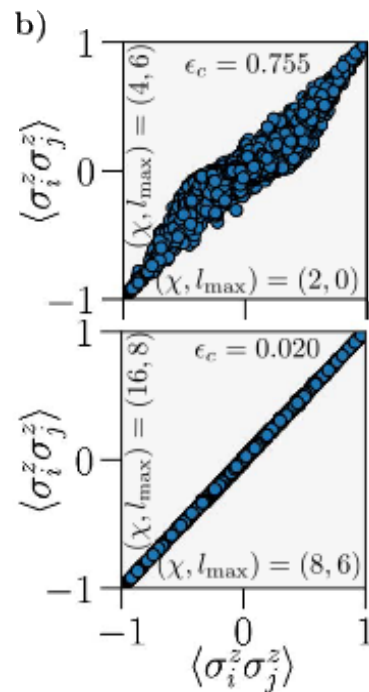
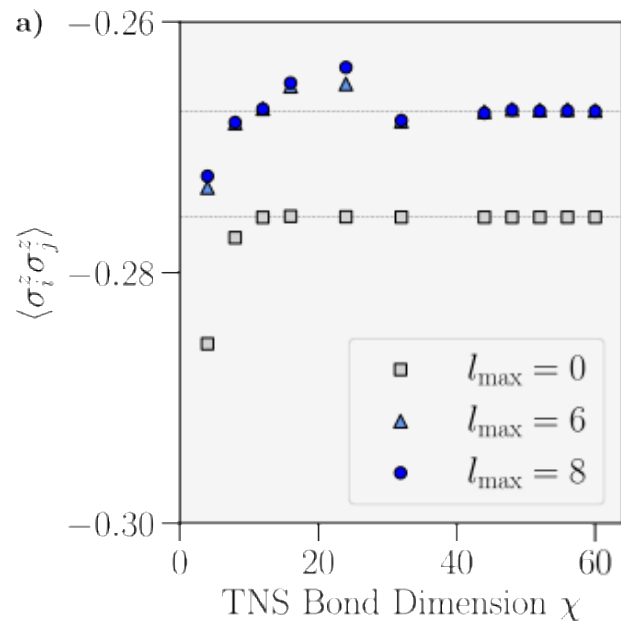


Quantum Annealer Loop Corrections $l \leq l_{\max}$



Disordered quantum dynamics in two and three dimensions

192 qubit diamond lattice, annealing time of 7ns



Emerging
convergence of
correlations

$$c_{i,j} = \langle \sigma_i^z \sigma_j^z \rangle$$

$$\epsilon_c = \sqrt{\frac{\sum_{i>j} (c_{i,j} - \tilde{c}_{i,j})^2}{\sum_{i>j} \tilde{c}_{i,j}^2}}$$

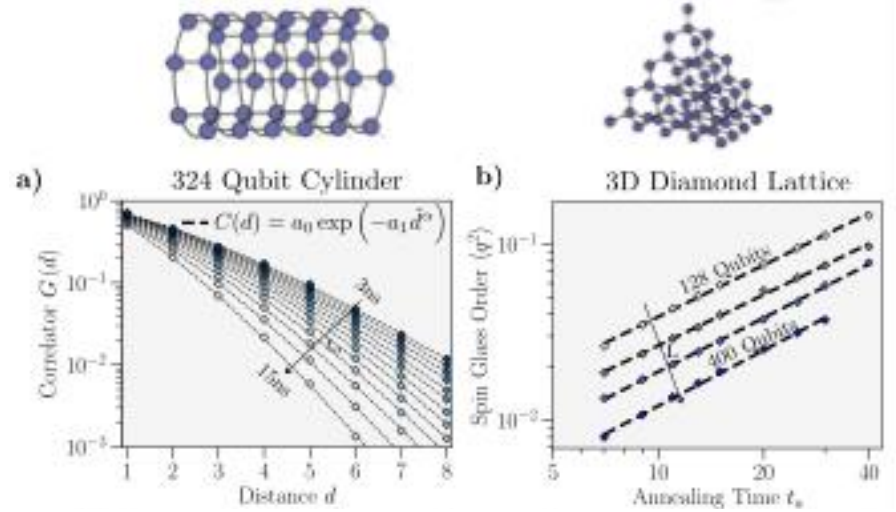
Exposing the Universal Physics

$$\xi \sim \tau_a^{\frac{1}{\mu}}$$

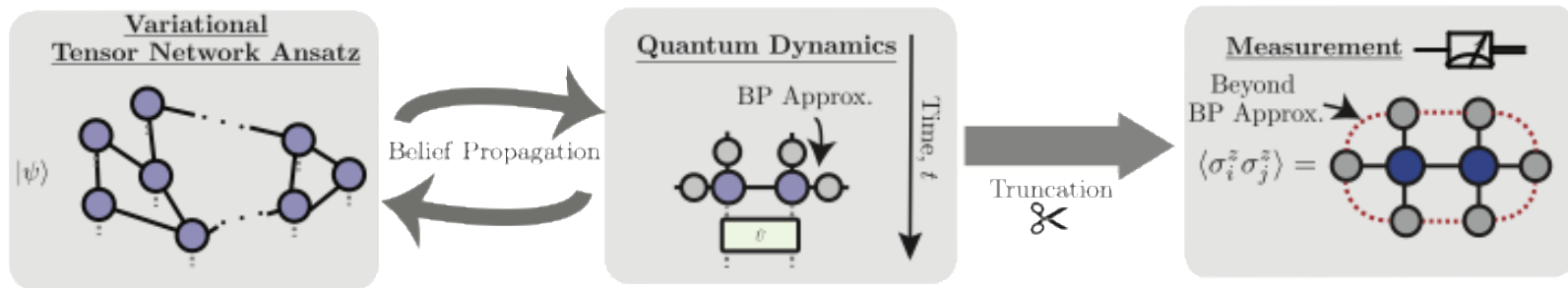
Correlation length

Annealing time / inverse velocity

Kibble Zurek exponent



Software – TNQS.jl



Software

TensorNetworkQuantumSimulator Public

- Julia package built on top of ITensors.jl and NamedGraphs.jl for quantum simulation:
<https://github.com/JoeyT1994/TensorNetworkQuantumSimulator>
- Full GPU Support via CUDA.jl
- Qiskit Wrapper Available: <https://github.com/qiskit-community/qiskit-tnqs>

Quantum computing companies + TNQS.jl



TensorNetworkQuantumSimulator Public

About



Package built on top of ITensors.jl and NamedGraphs.jl for quantum simulation with tensor networks (TNs) of near-arbitrary geometry

Readme

MIT license

Activity

82 stars

3 watching

19 forks

Releases 32

v0.4.1 Latest
yesterday



Quantum Computing Companies using
TensorNetworkQuantumSimulator.jl



PASQAL



Google AI
Quantum

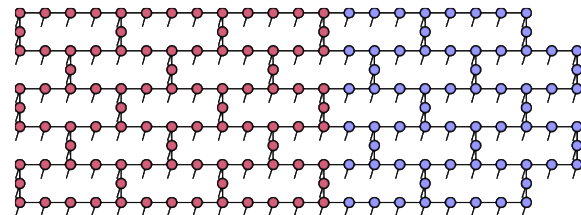


- (Pasqal) Vovrosh et al, Physical Review Research 8 023311 (2026)
- (Google) Bermejo et al, arXiv: 2604.15427 (2026)
- (IBM) Jaderberg et al, arXiv:2606.15983 (2026)
- (Pasqal + Flatiron) Forthcoming : “Exploring the relaxation landscape of a 2D Quantum magnet on a 256 qubit quantum processor” (2026)

GPU Acceleration

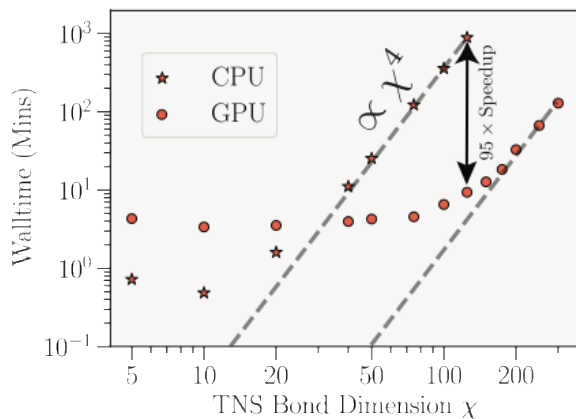


$$|\psi\rangle =$$

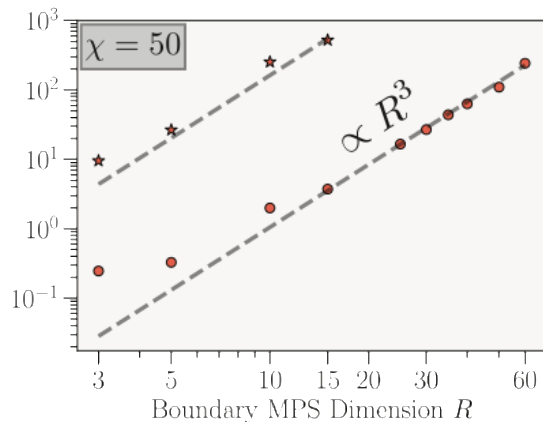


Inference methods on 2/3D Tensor Networks gain massive benefits from GPU hardware

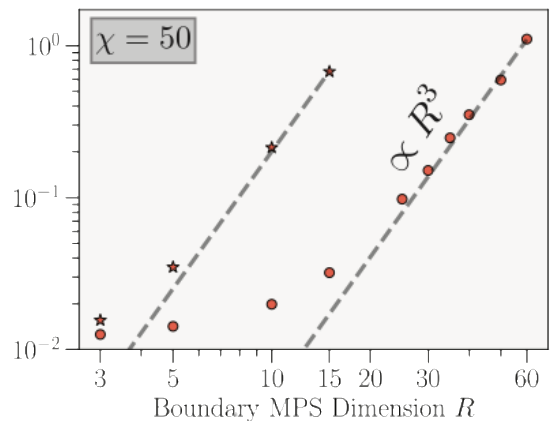
Circuit Evolution $|\psi_0\rangle \rightarrow C|\psi_0\rangle$



Direct Contraction of $\langle\psi|\psi\rangle$



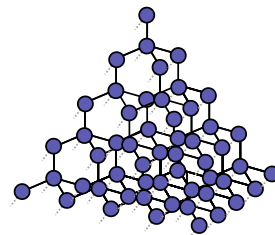
Generating Samples $x \sim |\langle x|\psi\rangle|^2$



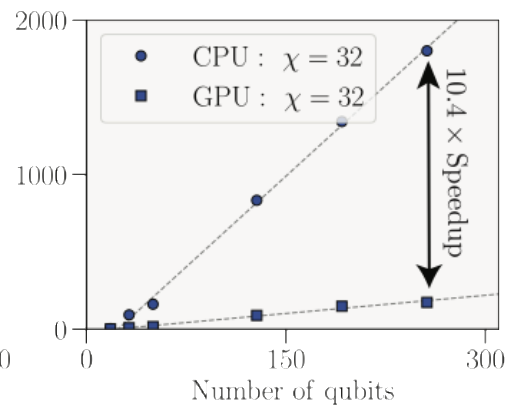
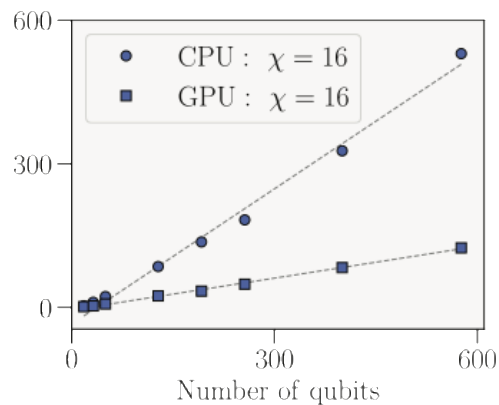
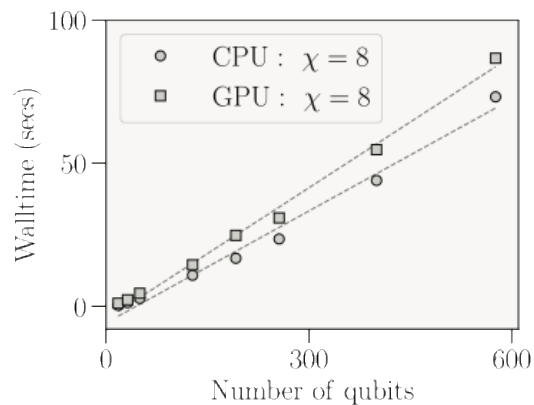
GPU Acceleration



$$|\psi\rangle =$$



Loop Corrected BP Results for Quantum Critical Dynamics:



Generalized Belief Propagation

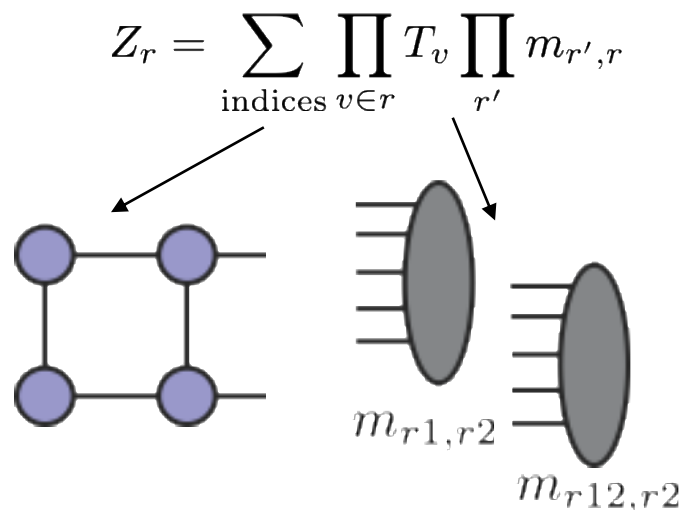
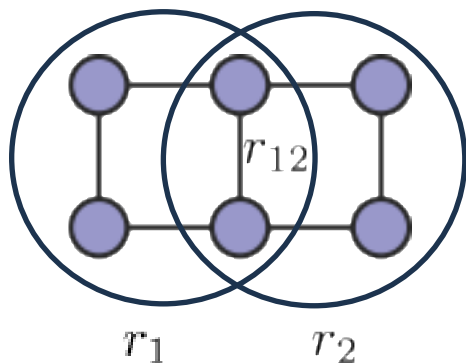
Can formulate a more general region-based approximation to the free energy of a tensor network based on the 'Kikuchi' free energy

**Bethe free energy, Kikuchi approximations,
and belief propagation algorithms**

Jonathan S. Yedidia, William T. Freeman, Yair Weiss

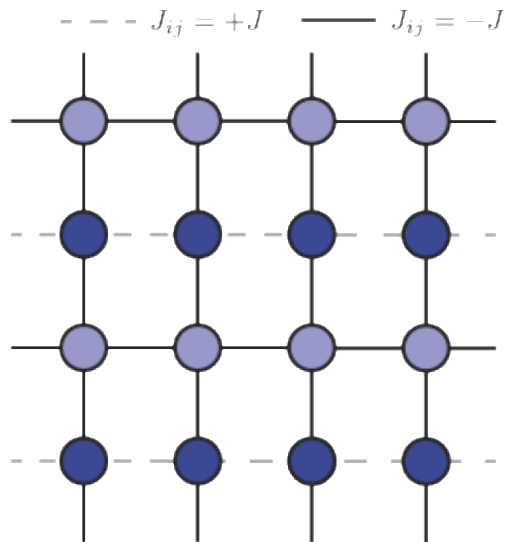
TR2001-16 May 2001

$$F_{\text{GBP}} = \sum_r c_r \log(Z_r) \approx \log(Z)$$

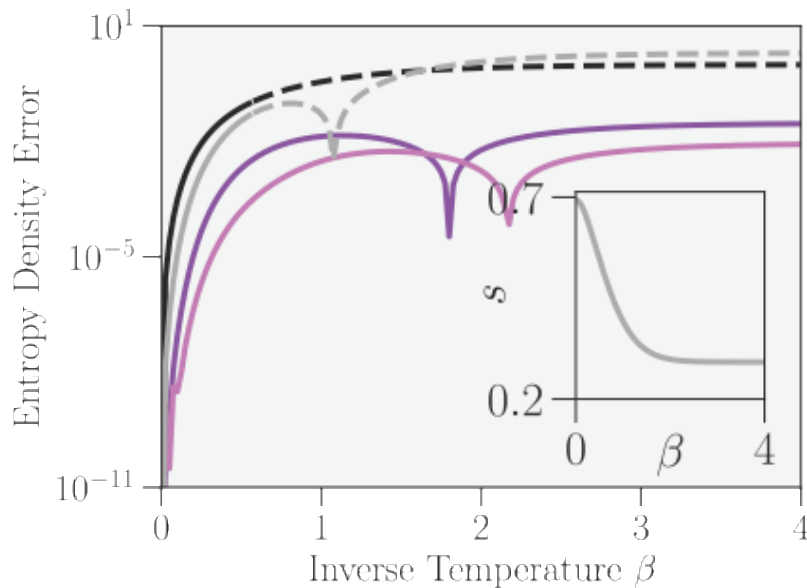


Correcting Belief Propagation – Generalized Belief Propagation

Impressive results on frustrated models

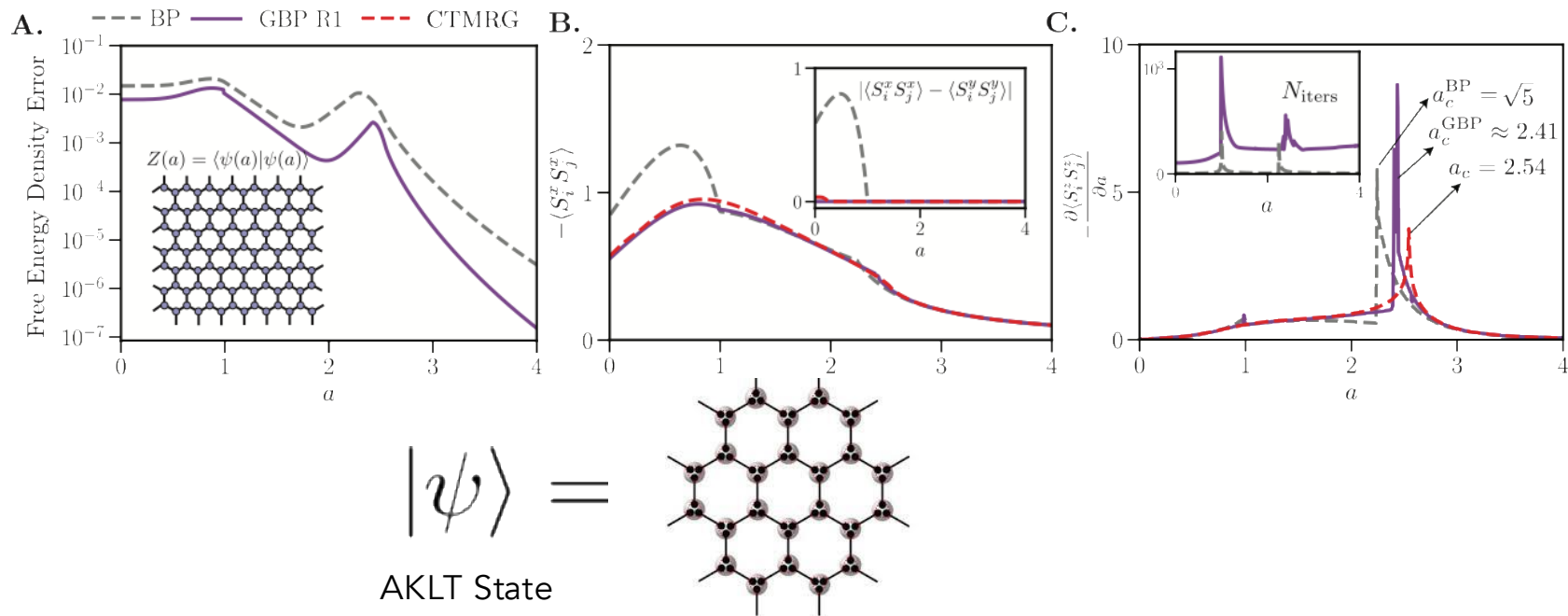


$$H = - \sum_{\langle i,j \rangle} J_{i,j} s_i s_j$$



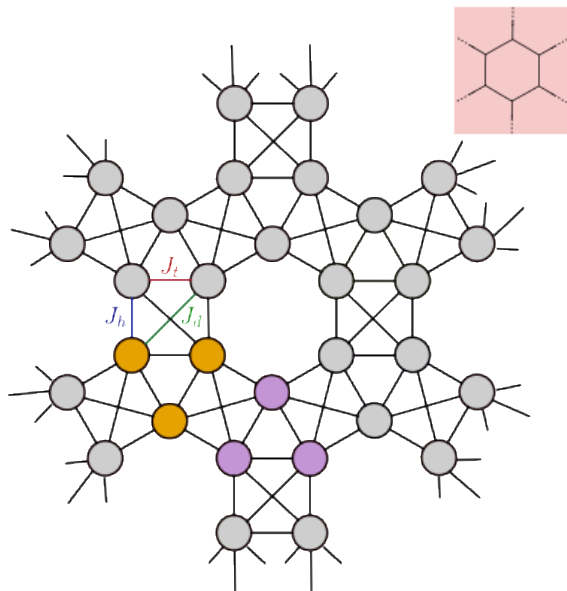
Correcting Belief Propagation – Generalized Belief Propagation

And on exotic quantum states...



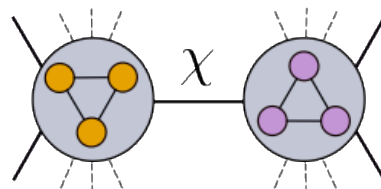
Frustrated Magnetism

$$H = \sum_{k \in \{t, h, d\}} \sum_{\langle ij \rangle} J_k \mathbf{S}_i \cdot \mathbf{S}_j + h \sum_i s_i^z$$



Ruby Lattice Structure

iTNS Ansatz



$$\sqrt{\rho(T)} = \exp \left(-\frac{\beta}{2} H \right) \mathcal{I}$$

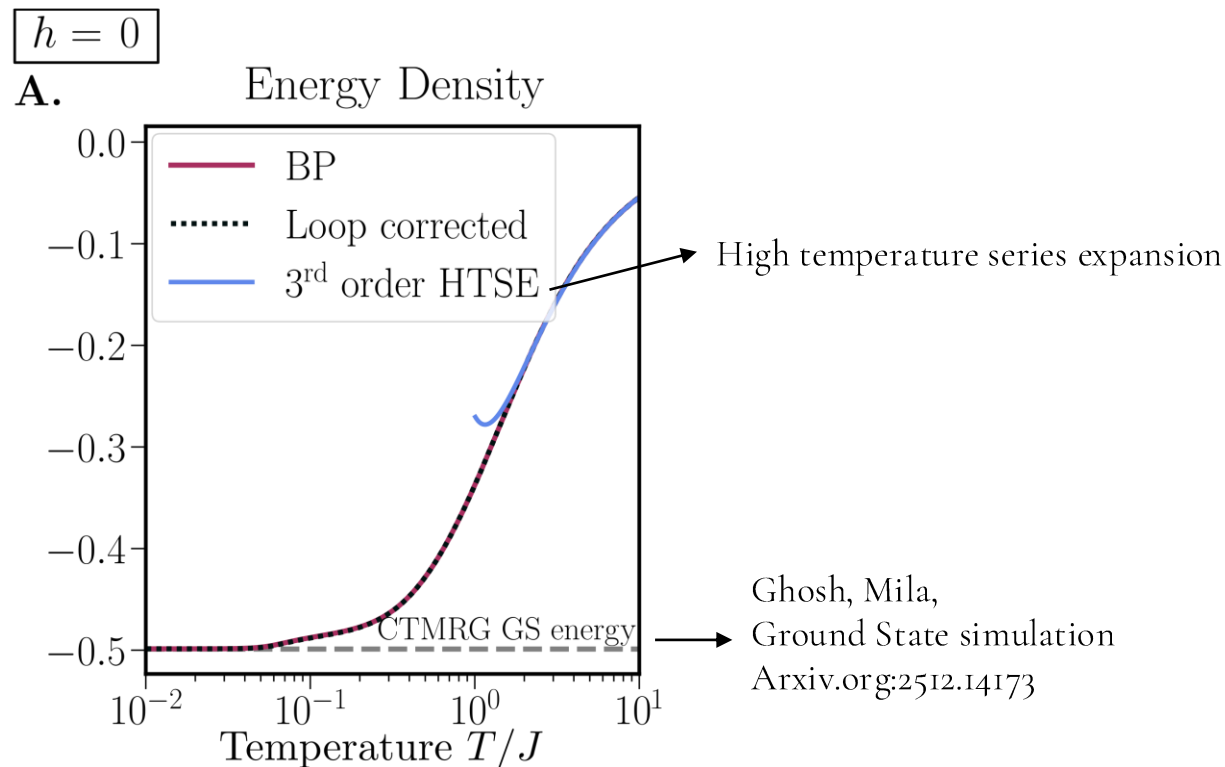
- Form the square root density matrix by using gate-based imaginary time evolution to cool down an initial identity matrix
- Take measurements via derivatives of the BP / loop corrected free energy
- Use GPUs to reach unprecedented bond dimensions



Antonio Mello
(CCQ Grad Scholar)

Frustrated Magnetism

BP handshakes with other methods at high and low temperatures



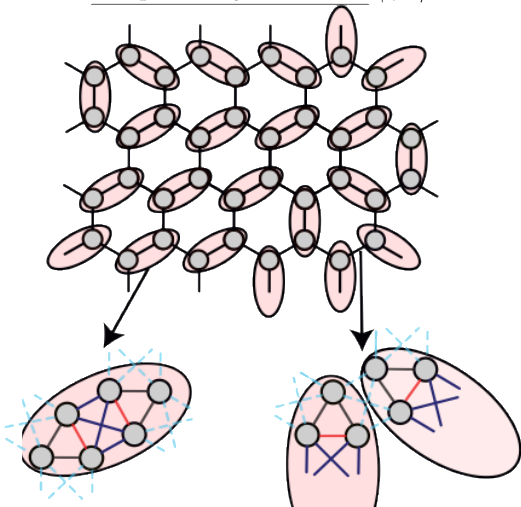
Frustrated Magnetism

Reveals an exotic, exponentially degenerate low energy manifold

B. Simplex Liquid State

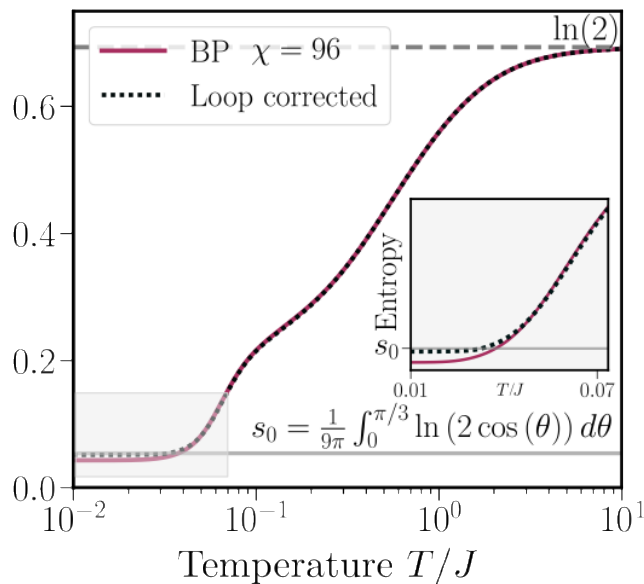
$$\lim_{T \rightarrow 0} \rho(T) = \frac{1}{|D|} \sum_{d \in D} |\psi_d\rangle \langle \psi_d|$$

Simplex Crystal State $|\psi_d\rangle$



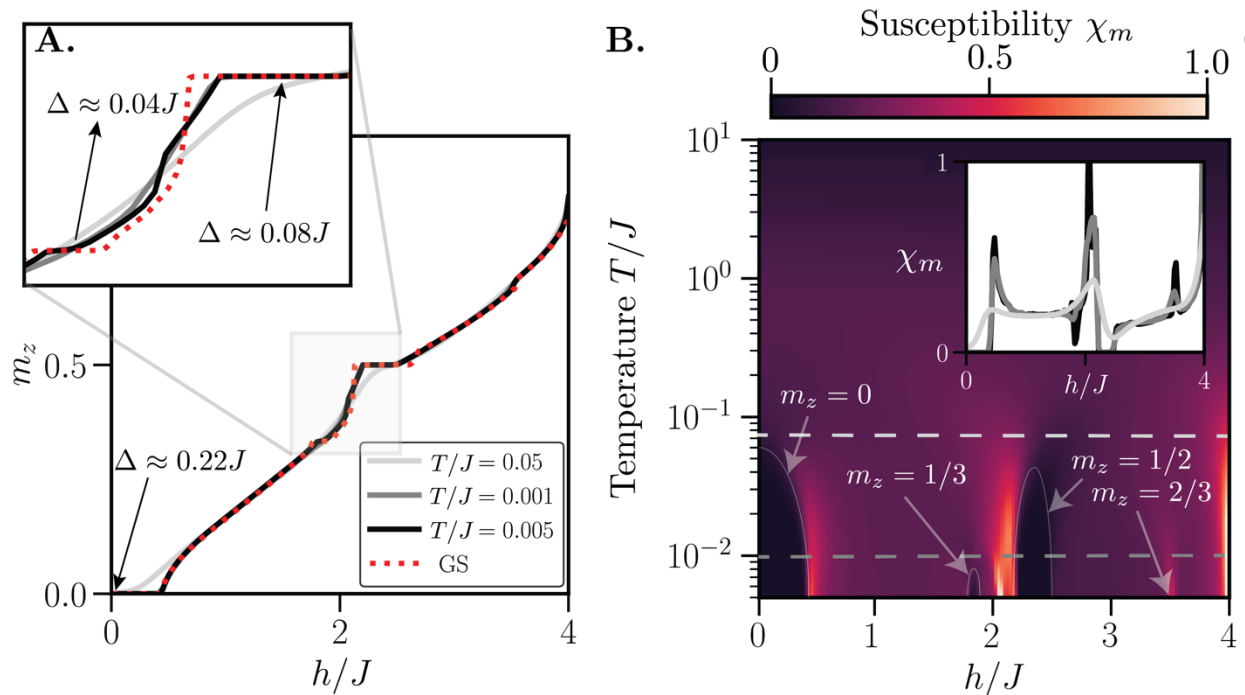
C.

Entropy Density



Frustrated Magnetism

Finite field reveals a low temperature phase diagram of magnetic plateaus hosting liquid states



The Future

- (Software): Continue to develop out our state-of-the-art tensor network software built around BP
- (Software + Algorithms): Exploit parallelism latent in these methods to push the simulation boundary
- (Algorithms): Develop new algorithms to correct BP and contract tensor networks
- (Algorithms): Utilize adaptive time stepping and complex time to push BP-based quantum dynamics further, see arXiv:2606.02930 (2026) and PRB 109 235110 (2025)
- (Physics): Work with experimentalists and quantum computing companies on benchmarking quantum simulators
- (Physics): Explore doped fermionic models at zero and finite temperature with fermionic tensor networks + BP
- (Physics): Study three-dimensional frustrated magnetic systems, see arXiv: 2606.07609

Thank you

Collaborators



Matt Fishman



Miles Stoudenmire



Antonio Mello



Dries Sels



Manuel Rudolph



Hilbert Kappen



Grace Sommers



Lukas Devos



Jack Dunham



IBM, Pasqal, Algorithmiq quantum teams